Total No. of Printed Pages—7

HS/XII/A. Sc. Com/M/14

2 0 1 4

MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Find the principal value of $\tan^{-1}(\sqrt{3}) - \text{cosec}^{-1}(-2)$. 2

2. Let N be set of all natural numbers and let R be a relation in N defined by $R = \{(a, b) : a \text{ is a factor of } b\}$. Show that R is reflexive and transitive. 2

3. Let

$$A \quad \begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$$

and $A^2 \quad KA$. Then find the value of K. 2

4. If

$$X \quad \begin{matrix} 3 & 4 \\ 1 & 2 \\ 0 & 1 \end{matrix} \quad \text{and} \quad Y \quad \begin{matrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{matrix}$$

then find $X \quad Y$. 2

5. Find the value of k if the function

$$f(x) \quad \begin{matrix} 3k \quad 2x, & \text{when } x \quad 1 \\ 2x \quad 1\ , & \text{when } x \quad 1 \end{matrix}$$

is continuous at $x \quad 1$. 2

6. If $x^y \quad e^{x \quad y}$, prove that

$$\frac{dy}{dx} \quad \frac{\log x}{(1 \quad \log x)^2}$$ 2

7. Evaluate

$$e^x(\tan x \quad 1)\sec x\, dx$$ 2

8. Let

$$f(x) \quad \frac{4x \quad 3}{6x \quad 4}, \quad x \quad \frac{2}{3}$$

Show that $(f \circ f)(x) \quad x$. 2

9. Form the differential equation representing the family of curves $y \quad ae^{2x} \quad be^{\ 2x}$ where a and b are arbitrary constants. 2

10. Evaluate

$$ {}_{2}^{1} |2x \quad 1|\,dx $$ 2

11. Prove that

$$ {}_{0}^{/2} \frac{\cos x}{\sin x \quad \cos x}\,dx \quad \frac{}{4} $$ 2

12. Let $\vec{a} \quad 2\hat{i} \quad 3\hat{j} \quad 2\hat{k}$ and $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$. Find the projection of $\vec{b}$ on $\vec{a}$. 2

13. Find the angle between the lines

$$ \frac{x \quad 1}{1} \quad \frac{y \quad 4}{1} \quad \frac{z \quad 5}{2} \text{ and } \frac{x \quad 3}{3} \quad \frac{y \quad 2}{5} \quad \frac{z \quad 5}{4} $$ 2

14. Find the distance between the parallel planes

$$ 2x \quad 3y \quad 4z \quad 4 \text{ and } 4x \quad 6y \quad 8z \quad 12 $$ 2

15. Find $P(A \quad B)$ when $2P(A) \quad P(B) \quad \frac{5}{13}$ and $P(A|B) \quad \frac{2}{5}$. 2

SECTION—B

16. Prove that

$$\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\quad x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$ 4

17. Using properties of determinant, prove that

$$\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix}=4abc$$ 4

18. Verify Rolle's theorem for the function

$$f(x)=\sin x+\cos x, \text{ in } \left[0,\frac{\pi}{2}\right]$$ 4

19. Find the equation of tangent to the curve $2x^2+y=7$ which is parallel to the line $4x-y+3=0$. 4

Or

The volume of a cube is increasing at the rate of 14 cm^3/sec. How fast is its surface area increasing at the instant when the length of an edge of the cube is 24 cm?

20. Solve the homogeneous differential equation

$$y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$$ 4

21. If

$$x = 2\cos\theta - \cos 2\theta \text{ and } y = 2\sin\theta - \sin 2\theta$$

find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{2}$. 4

22. Prove that

$$\int_0^{\pi/4} \log(1 + \tan x)\, dx = \frac{\pi}{8} \log 2$$ 4

Or

Evaluate

$$\int \frac{(x^2 + 1)e^x}{(x + 1)^2}\, dx$$

23. Solve the differential equation

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{(x^2 - 1)}$$ 4

24. Show that the lines

$$\frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4} \text{ and } \frac{x - 4}{5} = \frac{y - 1}{2} = z$$

intersect each other. Also find the point of intersection. 4

25. Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z - 4 = 0$. 4

SECTION—C

26. If

$$A \quad \begin{matrix} 2 & 3 & 5 \\ 3 & 2 & 4 \\ 1 & 1 & 2 \end{matrix}$$

find $A^{\;1}$. Using $A^{\;1}$, solve the following system of equations :

$$\begin{matrix} 2x & 3y & 5z & 11 \\ 3x & 2y & 4z & 5 \\ x & y & 2z & 3 \end{matrix}$$

6

27. Find the area of the region bounded by the curves $x^2 \quad y$ and $y^2 \quad x$. 6

28. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ times of the volume of the sphere. 6

Or

Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{\;1} \frac{1}{3}$.

29. An insurance company insured 2000 bike drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a bike, a car and a truck are ·01, ·03 and ·15 respectively. One of the insured persons meets with an accident. What is the probability that he is a bike driver? 6

30. A furniture dealer deals only two types of item—tables and chairs. He has ₹ 1,000 to invest and space to store almost 60 pieces. A table costs ₹ 500 and a chair costs ₹ 200. He can sell a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assume that he can sell all items that he buys. Using linear programming, formulate the problem for maximum profit and solve it graphically. 6

Or

A company manufactures two types of toy—*A* and *B*. Toy *A* requires 4 minutes for cutting and 8 minutes for assembling and toy *B* requires 8 minutes for cutting and 8 minutes for assembling. There are 3 hours and 20 minutes available in a day for cutting and 4 hours for assembling. The profit on a piece of toy *A* is ₹ 50 and that on toy *B* is ₹ 60. How many toys of each type should be made daily to have maximum profit? Solve the problem graphically.

★★★