STD. X
Mathematics II
Geometry
Sixth Edition: March 2016

Salient Features
• Written as per the new textbook.
• Exhaustive coverage of entire syllabus.
• Topic–wise distribution of all textual questions and practice problems at the beginning of every chapter.
• Covers solutions to all textual exercises and problem set.
• Includes additional problems for practice.
• Indicative marks for all problems.
• Comprehensive solution to Question Bank.
• Constructions drawn with accurate measurements.
• Includes Board Question Papers of 2014, 2015 and March 2016.

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Preface

Geometry is the mathematics of properties, measurement and relationships of points, lines, angles, surfaces and solids. It is widely used in the fields of science, engineering, computers, architecture etc. It is a vast subject dealing with the study of properties, definitions, theorems, areas, perimeter, angles, triangles, mensuration, co-ordinates, constructions etc.

The study of Geometry requires a deep and intrinsic understanding of concepts. Hence, to ease this task, we bring to you “Std. X: Geometry”, a complete and thorough guide critically analysed and extensively drafted to boost the confidence of the students. The question answer format of this book helps the student to understand and grasp each and every concept thoroughly. The book is based on the new text book and covers the entire syllabus. At the beginning of every chapter, topic-wise distribution of all textual questions and practice problems has been provided for simpler understanding of different types of questions. The book contains answers to textual exercises, problems sets and Question bank. It also includes additional questions for practice. All the diagrams are neat and have proper labelling. The book has a unique feature that all the constructions are as per the scale. Another feature of the book is its layout which is attractive and inspires the student to read.

Marks are provided for each and every problem. However, marks mentioned are indicative and are subject to change as per Maharashtra State Board’s discretion.

There is always room for improvement and hence we welcome all suggestions and regret any errors that may have occurred in the making of this book.

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

Yours’ faithfully,
Publisher

MARKING SCHEME

Marking Scheme (for March 2014 exam and onwards)

<table>
<thead>
<tr>
<th>Written Exam</th>
<th>Marks</th>
<th>Time</th>
</tr>
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<tbody>
<tr>
<td>Algebra</td>
<td>40</td>
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</tr>
<tr>
<td>Geometry</td>
<td>40</td>
<td>2 hrs.</td>
</tr>
<tr>
<td>* Internal Assessment</td>
<td>20</td>
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</tr>
<tr>
<td><strong>Total</strong></td>
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</tr>
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* Internal Assessment

Home Assignment: 10 Marks

Test of multiple choice question: 10 Marks

Total 20 marks

5-5 Home assignment for Algebra and Geometry of 10 marks each would be given. Marks obtained out of 100 would be converted to marks out of 10.

Depending upon the entire syllabus, internal test for Algebra and Geometry with 20 marks each would be taken at the end of second semester. Marks obtained out of 40 would be converted to marks out of 10.
# ALGEBRA AND GEOMETRY

## Mark Wise Distribution of Questions

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<tr>
<th>Type of Questions</th>
<th>Marks</th>
<th>Marks with Option</th>
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<tbody>
<tr>
<td>6 sub questions of 1 mark each:</td>
<td>05</td>
<td>06</td>
</tr>
<tr>
<td>6 sub questions of 2 marks each:</td>
<td>08</td>
<td>12</td>
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<tr>
<td>5 sub questions of 3 marks each:</td>
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<td>15</td>
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<tr>
<td>3 sub questions of 4 marks each:</td>
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<td>3 sub questions of 5 marks each:</td>
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## Weightage to Types of Questions

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<th>Marks</th>
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<td>Very short answer</td>
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<tr>
<td>2</td>
<td>Short answer</td>
<td>27</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>Long answer</td>
<td>27</td>
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## Weightage to Objectives

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<th>Algebra Percentage marks</th>
<th>Geometry Percentage marks</th>
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<tr>
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<td>Knowledge</td>
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<td>15</td>
</tr>
<tr>
<td>2</td>
<td>Understanding</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>Application</td>
<td>60</td>
<td>50</td>
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<tr>
<td>4</td>
<td>Skill</td>
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## Unit wise Distribution: Algebra

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<td>1</td>
<td>Arithmetic Progression</td>
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<tr>
<td>2</td>
<td>Quadratic equations</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>Linear equation in two variables</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Probability</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Statistics – I</td>
<td>06</td>
</tr>
<tr>
<td>6</td>
<td>Statistics – II</td>
<td>08</td>
</tr>
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## Unit wise Distribution: Geometry

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<td>1</td>
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<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Circle</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Geometric Constructions</td>
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</tr>
<tr>
<td>4</td>
<td>Trigonometry</td>
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<tr>
<td>5</td>
<td>Co-ordinate Geometry</td>
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<tr>
<td>6</td>
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<td><strong>Total:</strong></td>
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<td>Q.7 (iii.), 20</td>
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<td>Q.13, 14</td>
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</table>
Similarity of triangles

For a given one-to-one correspondence between the vertices of two triangles, if
i. their corresponding angles are congruent and
ii. their corresponding sides are in proportion then the correspondence is known as similarity and the two triangles are said to be similar.

In the figure, for correspondence ABC ↔ PQR,
i. \( \angle A \cong \angle P, \angle B \cong \angle Q, \angle C \cong \angle R \)
ii. \( \frac{AB}{PQ} = \frac{2}{3}, \frac{BC}{QR} = \frac{6}{9} = \frac{2}{3}, \frac{AC}{PR} = \frac{4}{6} = \frac{2}{3} \)
i.e., \( \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \)

Hence, \( \triangle ABC \) and \( \triangle PQR \) are similar triangles and are symbolically written as \( \triangle ABC \sim \triangle PQR \).

Test of similarity of triangles

1. **S–S–S test of similarity:**

   For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if the sides of one triangle are proportional to the corresponding sides of the other triangle.

   In the figure,
   \[ \frac{AB}{PQ} = \frac{1}{2}, \frac{BC}{QR} = \frac{3}{6} = \frac{1}{2}, \frac{AC}{PR} = \frac{2}{4} = \frac{1}{2} \]
   \[ \therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \]
   \[ \therefore \triangle ABC \sim \triangle PQR \quad ---- \text{[By S–S–S test of similarity]} \]

2. **A–A–A test of similarity [A–A test]:**

   For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if the angles of one triangle are congruent to the corresponding angles of the other triangle.

   In the figure,
   if \( \angle A \cong \angle P, \angle B \cong \angle Q, \angle C \cong \angle R \)
   then \( \triangle ABC \sim \triangle PQR \quad ---- \text{[By A–A–A test of similarity]} \)

**Note:** A–A–A test is verified same as A–A test of similarity.

3. **S–A–S test of similarity:**

   For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if two sides of a triangle are proportional to the two corresponding sides of the other triangle and the corresponding included angles are also congruent.

   In the figure,
   \[ \frac{AB}{PQ} = \frac{1}{3}, \frac{BC}{QR} = \frac{2}{6} = \frac{1}{3} \]
   \[ \therefore \frac{AB}{PQ} = \frac{BC}{QR} \text{ and } \angle B \cong \angle Q \]
   \[ \therefore \triangle ABC \sim \triangle PQR \quad ---- \text{[By S–A–S test of similarity]} \]
Chapter 01: Similarity

Converse of the test for similarity:

i. **Converse of S–S–S test:**
   If two triangles are similar, then the corresponding sides are in proportion.
   If \( \triangle ABC \sim \triangle PQR \) then,
   \[
   \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad ---- \text{[Corresponding sides of similar triangles]}
   \]

ii. **Converse of A–A–A test:**
   If two triangles are similar, then the corresponding angles are congruent.
   If \( \triangle ABC \sim \triangle PQR \),
   then \( \angle A \cong \angle P, \angle B \cong \angle Q \) and \( \angle C \cong \angle R \) ---- \text{[Corresponding angles of similar triangles]}

Note: ‘Corresponding angles of similar triangles’ can also be written as c.a.s.t.
‘Corresponding sides of similar triangles’ can also be written as c.s.s.t.

### 1.1 Properties of the ratios of areas of two triangles

**Property – I**

The ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights.  

\[ \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC \times AD}{QR \times PS} \]

**Given:** In \( \triangle ABC \) and \( \triangle PQR \), seg AD \( \perp \) seg BC, B–D–C,  
seg PS \( \perp \) ray RQ, S–Q–R

**To prove that:**  
\[ \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{BC \times AD}{QR \times PS} \]

**Proof:**

\[
\begin{align*}
A(\triangle ABC) &= \frac{1}{2} \times BC \times AD \quad ---- (i) \\
A(\triangle PQR) &= \frac{1}{2} \times QR \times PS \quad ---- (ii) \\
\text{Dividing (i) by (ii), we get} \quad \frac{A(\triangle ABC)}{A(\triangle PQR)} &= \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS} \\
\therefore \quad \frac{A(\triangle ABC)}{A(\triangle PQR)} &= \frac{BC \times AD}{QR \times PS}
\end{align*}
\]

**For Understanding**

**When do you say the triangles have equal heights?**

We can discuss this in three cases.

**Case – I**

In the adjoining figure,  
segments AD and PS are the corresponding heights of \( \triangle ABC \) and \( \triangle PQR \) respectively.

If AD = PS, then \( \triangle ABC \) and \( \triangle PQR \) are said to have equal heights.

**Case – II**

In the adjoining figure, \( \triangle ABC \) and \( \triangle XYZ \) have their one vertex on one of the parallel lines and the other two vertices lie on the other parallel line. Hence the two triangles are said to lie between the same parallel lines and are said to have equal heights.
**Case – III**
In the adjoining figure, \( \triangle ABC, \triangle ACD \) and \( \triangle ABD \) have a common vertex \( A \) and the sides opposite to vertex \( A \) namely, \( BC, CD \) and \( BD \) respectively of these triangles lie on the same line. Hence, \( \triangle ABC, \triangle ACD \) and \( \triangle ABD \) are said to have equal heights and \( BC, CD \) and \( BD \) are their respective bases.

**Property – II**
The ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.

Example:
\( \triangle ABC \) and \( \triangle DCB \) have a common base \( BC \).
\[ \frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{AP}{DQ} \]

**Property – III**
The ratio of areas of two triangles having equal height is equal to the ratio of their corresponding bases.

Example:
\( \triangle ABC, \triangle ACD \) and \( \triangle ABD \) have a common vertex \( A \) and their sides opposite to vertex \( A \) namely, \( BC, CD, BD \) respectively lie on the same line. Hence they have equal heights. Here, \( AP \) is common height.
\[ \frac{A(\triangle ABC)}{A(\triangle ACD)} = \frac{BC}{CD}, \quad \frac{A(\triangle ABD)}{A(\triangle ACD)} = \frac{BD}{CD} \]

**Property – IV**
Areas of two triangles having equal bases and equal heights are equal.

Example:
\( \triangle ABD \) and \( \triangle ACD \) have a common vertex \( A \) and their sides opposite to vertex \( A \) namely, \( BD \) and \( DC \) respectively lie on the same line. Hence the triangles have equal heights. Also their bases \( BD \) and \( DC \) are equal.
\[ A(\triangle ABD) = A(\triangle ACD) \]

### Exercise 1.1

1. In the adjoining figure, seg \( BE \perp \) seg \( AB \) and seg \( BA \perp \) seg \( AD \).
   If \( BE = 6 \) and \( AD = 9 \), find \( \frac{A(\triangle ABE)}{A(\triangle BAD)} \). [Oct 14, July 15] [1 mark]

**Solution:**
\[ \frac{A(\triangle ABE)}{A(\triangle BAD)} = \frac{BE}{AD} \quad \text{[Ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.]} \]
\[ \therefore \quad \frac{A(\triangle ABE)}{A(\triangle BAD)} = \frac{6}{9} \]
\[ \therefore \quad \frac{A(\triangle ABE)}{A(\triangle BAD)} = \frac{2}{3} \]
2. In the adjoining figure, seg SP \perp \text{side YK} and seg YT \perp \text{seg SK}. If SP = 6, YK = 13, YT = 5 and TK = 12, then find \( A(\triangle SYK) : A(\triangle YTK) \).  

Solution:

\[
\frac{A(\triangle SYK)}{A(\triangle YTK)} = \frac{YK \times SP}{TK \times YT} \]

\[
\frac{A(\triangle SYK)}{A(\triangle YTK)} = \frac{13 \times 6}{12 \times 5} \]

\[
\frac{A(\triangle SYK)}{A(\triangle YTK)} = \frac{13}{10} \]

\[
A(\triangle SYK) : A(\triangle YTK) = 13 : 10
\]

3. In the adjoining figure, RP : PK = 3 : 2, then find the values of the following ratios:

i. \( A(\triangle TRP) : A(\triangle TPK) \)  

ii. \( A(\triangle TRK) : A(\triangle TPK) \)  

iii. \( A(\triangle TRP) : A(\triangle TRK) \)  

Solution:

\[
RP : PK = 3 : 2 \quad \text{[Given]}
\]

Let the common multiple be \( x \).

\[
RP = 3x, \quad PK = 2x \quad \text{[From (i)]}
\]

\[
RK = RP + PK \quad \text{[R–P–K]}
\]

\[
RK = 3x + 2x \quad \text{[From (i)]}
\]

\[
RK = 5x
\]

i. \( \frac{A(\triangle TRP)}{A(\triangle TPK)} = \frac{RP}{PK} \quad \text{[Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]} 
\]

\[
\frac{A(\triangle TRP)}{A(\triangle TPK)} = \frac{3x}{2x} \quad \text{[From (i)]}
\]

\[
\frac{A(\triangle TRP)}{A(\triangle TPK)} = \frac{3}{2}
\]

\[
A(\triangle TRP) : A(\triangle TPK) = 3 : 2
\]

ii. \( \frac{A(\triangle TRK)}{A(\triangle TPK)} = \frac{RK}{PK} \quad \text{[Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]} 
\]

\[
\frac{A(\triangle TRK)}{A(\triangle TPK)} = \frac{5x}{2x} \quad \text{[From (i) and (ii)]}
\]

\[
\frac{A(\triangle TRK)}{A(\triangle TPK)} \frac{5}{2}
\]

\[
A(\triangle TRK) : A(\triangle TPK) = 5 : 2
\]

iii. \( \frac{A(\triangle TRP)}{A(\triangle TRK)} = \frac{RP}{RK} \quad \text{[Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]} 
\]

\[
\frac{A(\triangle TRP)}{A(\triangle TRK)} = \frac{3x}{5x} \quad \text{[From (i) and (ii)]}
\]

\[
\frac{A(\triangle TRP)}{A(\triangle TRK)} = \frac{3}{5}
\]

\[
A(\triangle TRP) : A(\triangle TRK) = 3 : 5
\]
4. The ratio of the areas of two triangles with the common base is $6 : 5$. Height of the larger triangle is $9$ cm. Then find the corresponding height of the smaller triangle. [Mar 15] [3 marks]

Solution:
Let $A_1$ and $A_2$ be the areas of larger triangle and smaller triangle respectively and $h_1$ and $h_2$ be their corresponding heights.

\[
\frac{A_1}{A_2} = \frac{6}{5} \quad \text{---- (i) [Given]}
\]

\[h_1 = 9 \quad \text{---- (ii) [Given]}\]

\[
\frac{A_1}{A_2} = \frac{h_1}{h_2} \quad \text{---- [Ratio of the areas of two triangles having equal base is equal to the ratio of their corresponding heights.]}
\]

\[\therefore \quad \frac{6}{5} = \frac{9}{h_2} \quad \text{---- [From (i) and (ii)]}\]

\[\therefore \quad h_2 = \frac{5 \times 9}{6} \]

\[\therefore \quad h_2 = 7.5 \text{ cm}\]

\[
\therefore \quad \text{The corresponding height of the smaller triangle is 7.5 cm.}
\]

5. In the adjoining figure, seg PR ⊥ seg BC, seg AS ⊥ seg BC and seg QT ⊥ seg BC. Find the following ratios: [3 marks]

i. \[\frac{A(\triangle ABC)}{A(\triangle PBC)}\]  
ii. \[\frac{A(\triangle ABS)}{A(\triangle ASC)}\]  
iii. \[\frac{A(\triangle PRC)}{A(\triangle BQT)}\]  
iv. \[\frac{A(\triangle BPR)}{A(\triangle CQT)}\]

Solution:

i. \[\frac{A(\triangle ABC)}{A(\triangle PBC)} = \frac{AS}{PR} \quad \text{---- [Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.]}\]

ii. \[\frac{A(\triangle ABS)}{A(\triangle ASC)} = \frac{BS}{SC} \quad \text{---- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]}\]

iii. \[\frac{A(\triangle PRC)}{A(\triangle BQT)} = \frac{RC \times PR}{BT \times QT} \quad \text{---- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]}\]

iv. \[\frac{A(\triangle BPR)}{A(\triangle CQT)} = \frac{BR \times PR}{CT \times QT} \quad \text{---- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]}\]

6. In the adjoining figure, seg DH ⊥ seg EF and seg GK ⊥ seg EF. If DH = 12 cm, GK = 20 cm and $A(\triangle DEF) = 300$ cm$^2$, then find

i. EF  
ii. $A(\triangle GEF)$  
iii. $A(\square DFGE)$ [3 marks]

Solution:

i. Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

\[\therefore \quad A(\triangle DEF) = \frac{1}{2} \times EF \times DH\]

\[\therefore \quad 300 = \frac{1}{2} \times EF \times 12 \quad \text{---- [Substituting the given values]}\]

\[\therefore \quad 300 = EF \times 6\]

\[\therefore \quad EF = \frac{300}{6}\]

\[\therefore \quad EF = 50 \text{ cm}\]
Chapter 01: Similarity

ii. \( \frac{A(\triangle DEF)}{A(\triangle GEF)} = \frac{DH}{GK} \)  

\[ \frac{300}{A(\triangle GEF)} = \frac{12}{20} \]

\[ \therefore 300 \times 20 = 12 \times A(\triangle GEF) \]

\[ \frac{300 \times 20}{12} = A(\triangle GEF) \]

\[ \therefore A(\triangle GEF) = \frac{300 \times 20}{12} \]

\[ \therefore A(\triangle GEF) = 500 \text{ cm}^2 \] ---- (i)

iii. \( A(\triangle DFGE) = A(\triangle DEF) + A(\triangle GEF) \) ---- [Area addition property]

\[ \therefore A(\triangle DFGE) = 300 + 500 \] ---- [From (i) and given]

\[ \therefore A(\triangle DFGE) = 800 \text{ cm}^2 \]

7. In the adjoining figure, seg ST || side QR. Find the following ratios. [3 marks]

i. \( \frac{A(\triangle PST)}{A(\triangle QST)} \)  

ii. \( \frac{A(\triangle PST)}{A(\triangle RST)} \)  

iii. \( \frac{A(\triangle QST)}{A(\triangle RST)} \)

**Solution:**

i. \( \frac{A(\triangle PST)}{A(\triangle QST)} = \frac{PS}{QS} \) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

ii. \( \frac{A(\triangle PST)}{A(\triangle RST)} = \frac{PT}{TR} \)

iii. \( \triangle QST \) and \( \triangle RST \) lie between the same parallel lines ST and QR.

\[ \therefore \text{Their heights are equal.} \]

\[ \text{Also ST is the common base.} \]

\[ \therefore \frac{A(\triangle QST)}{A(\triangle RST)} = 1 \] ---- [Areas of two triangles having common base and equal heights are equal.]

### 1.2 Basic Proportionality Theorem (B.P.T.)

If a line parallel to a side of a triangle intersects the other sides in two distinct points, then the line divides these sides in proportion.  

**Given:** In \( \triangle PQR \), line \( l \parallel \text{side QR}. \)

Line \( l \) intersects side PQ and side PR in points M and N respectively, such that P–M–Q and P–N–R.

**To Prove that:** \( \frac{PM}{PN} = \frac{MQ}{NR} \)

**Construction:** Draw seg QN and seg RM.

**Proof:**

In \( \triangle PMN \) and \( \triangle QMN \), where P–M–Q,

\[ \frac{A(\triangle PMN)}{A(\triangle QMN)} = \frac{PM}{MQ} \] ---- (i) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

In \( \triangle PMN \) and \( \triangle ARM \), where P–N–R,

\[ \frac{A(\triangle PMN)}{A(\triangle ARM)} = \frac{PN}{NR} \]

\[ A(\triangle QMN) = A(\triangle ARM) \]

\[ \therefore \frac{A(\triangle PMN)}{A(\triangle QMN)} = \frac{A(\triangle PMN)}{A(\triangle ARM)} \] ---- (ii) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

\[ \therefore \frac{PN}{NR} = \frac{PM}{MQ} \] ---- (iii) [Areas of two triangles having equal bases and equal heights are equal.]

\[ \therefore \frac{PM}{PN} = \frac{MQ}{NR} \] ---- (iv) [From (i), (ii) and (iii)]
Converse of Basic Proportionality Theorem:

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If line \( l \) intersects the side PQ and side PR of \( \triangle PQR \) in the points M and N respectively such that \( \frac{PM}{MQ} = \frac{PN}{NR} \), then line \( l \parallel \) side QR.

Applications of Basic Proportionality Theorem:

i. Property of intercepts made by three parallel lines on a transversal:
The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same three parallel lines. \[3\text{ marks}\]

Given: line \( l \parallel \) line m \parallel line n
The transversals x and y intersect these parallel lines at points A, B, C and P, Q, R respectively.

To Prove that: \( \frac{AB}{BC} = \frac{PQ}{QR} \)

Construction: Draw seg AR to intersect line m at point H.

Proof:

In \( \triangle ACR \),
seg BH \parallel side CR \hspace{1cm} \text{[Given]}

\( \therefore \frac{AB}{BC} = \frac{AH}{HR} \hspace{1cm} \text{[By B.P.T.]} \)

In \( \triangle ARP \),
seg HQ \parallel side AP \hspace{1cm} \text{[Given]}

\( \frac{QR}{PQ} = \frac{RH}{HA} \hspace{1cm} \text{[By B.P.T.]} \)

\( \therefore \frac{QR}{PQ} = \frac{AH}{HR} \hspace{1cm} \text{[By invertendo]} \)

\( \therefore \frac{AB}{BC} = \frac{PQ}{QR} \hspace{1cm} \text{[From (i) and (ii)]} \)

ii. Property of an angle bisector of a triangle:
In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides. \[Mar 15\] \[5\text{ marks}\]

Given: In \( \triangle ABC \), ray AD bisects \( \angle BAC \)

To Prove that: \( \frac{BD}{DC} = \frac{AB}{AC} \)

Construction: Draw a line parallel to ray AD, passing through point C. Extend BA to intersect the line at E.

Proof:

In \( \triangle BEC \),
seg AD \parallel side EC \hspace{1cm} \text{[By construction]}

\( \therefore \frac{BD}{DC} = \frac{AB}{AE} \hspace{1cm} \text{[By B.P.T.]} \)

line AD \parallel line EC on transversal BE

\( \therefore \angle BAD \cong \angle AEC \hspace{1cm} \text{[Corresponding angles]} \)

line AD \parallel line EC on transversal AC.

\( \therefore \angle CAD \cong \angle ACE \hspace{1cm} \text{[Alternate angles]} \)

Also, \( \angle BAD \cong \angle CAD \hspace{1cm} \text{[\because Ray AD bisects \( \angle BAC \)]} \)

\( \therefore \angle AEC \cong \angle ACE \hspace{1cm} \text{[From (ii), (iii) and (iv)]} \)
In \( \Delta AEC \),
\[ \angle AEC \cong \angle ACE \]  
---- [From (v)]
\[ \therefore \ AE = AC \]  
---- (vi) [Sides opposite to congruent angles]
\[ \therefore \ BD = AB \]  
----- [From (i) and (vi)]

**Exercise 1.2**

1. Find the values of \( x \) in the following figures, if line \( l \) is parallel to one of the sides of the given triangles.

   [Oct 12, Mar 13] [1 mark each]

   ![Diagram](image)

**Solution:**

i. In \( \Delta ABC \),
line \( l \parallel \text{side BC} \)  
---- [Given]
\[ \therefore \ AP = \frac{AY}{PB} = \frac{YC}{X} \]  
---- [By B.P.T.]
\[ \therefore \ \frac{3}{6} = \frac{5}{x} \]
\[ \therefore \ x = \frac{6 \times 5}{3} \]
\[ \therefore \ x = \boxed{10 \text{ units}} \]

ii. In \( \Delta RST \),
line \( l \parallel \text{side TR} \)  
---- [Given]
\[ \frac{SP}{PT} = \frac{SQ}{QR} \]  
---- [By B.P.T.]
\[ \therefore \ \frac{x}{4.5} = \frac{1.3}{3.9} \]
\[ \therefore \ x = \frac{1.3 \times 4.5}{3.9} \]
\[ \therefore \ x = \frac{13 \times 45}{39 \times 10} \]
\[ \therefore \ x = \boxed{1.5 \text{ units}} \]

iii. In \( \Delta LMN \),
line \( l \parallel \text{side LN} \)  
---- [Given]
\[ \frac{MP}{PL} = \frac{MQ}{QN} \]  
---- [By B.P.T.]
\[ \therefore \ \frac{8}{2} = \frac{x}{3} \]
\[ \therefore \ \frac{3 \times 8}{2} = x \]
\[ \therefore \ x = 3 \times 4 \]
\[ \therefore \ x = \boxed{12 \text{ units}} \]
2. E and F are the points on the side PQ and PR respectively of \( \triangle PQR \). For each of the following cases, state whether \( EF \parallel QR \). [2 marks each]

i. PE = 3.9 cm, EQ = 1.3 cm, PF = 3.6 cm and FR = 2.4 cm.
ii. PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.
iii. PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

**Solution:**

i. \[
\frac{PE}{EQ} = \frac{3.9}{1.3} = \frac{3}{1} \quad ---- (i) \\
\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} \quad ---- (ii)
\]

\[
\therefore \text{In } \triangle PQR, \quad \frac{PE}{EQ} \neq \frac{PF}{FR} \quad ---- [\text{From (i) and (ii)}]
\]

\[
\therefore \text{seg } EF \text{ is not parallel to seg } QR.
\]

ii. \[
\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9} \quad ---- (i) \\
\frac{PF}{FR} = \frac{8}{9} \quad ---- (ii)
\]

\[
\text{In } \triangle PQR, \quad \frac{PE}{QE} = \frac{PF}{FR} \quad ---- [\text{From (i) and (ii)}]
\]

\[
\therefore \text{seg } EF \parallel \text{seg } QR \quad ---- [\text{By converse of B.P.T.}]
\]

iii. \[
EQ + PE = PQ \quad ---- [P-E-Q] \\
\therefore \quad EQ = PQ - PE = 1.28 - 0.18 = 1.10
\]

\[
FR + PF = PR \quad ---- [P-F-R] \\
\therefore \quad FR = PR - PF = 2.56 - 0.36 = 2.20
\]

\[
\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55} \quad ---- (i) \\
\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55} \quad ---- (ii)
\]

\[
\text{In } \triangle PQR, \quad \frac{PE}{EQ} = \frac{PF}{FR} \quad ---- [\text{From (i) and (ii)}]
\]

\[
\therefore \text{seg } EF \parallel \text{side } QR \quad ---- [\text{By converse of B.P.T.}]
\]

3. In the adjoining figure, point Q is on the side MP such that MQ = 2 and MP = 5.5. Ray NQ is the bisector of \( \angle MNP \) of \( \triangle MNP \). Find MN : NP. [2 marks]

**Solution:**

\[
QP + MQ = MP \quad ---- [M-Q-P]
\]

\[
\therefore \quad QP = 5.5 - 2 = 3.5
\]

\[
\text{In } \triangle MNP, \quad \text{ray } NQ \text{ is the angle bisector of } \angle MNP \quad ---- [\text{Given}]
\]

\[
\therefore \quad \frac{MN}{NP} = \frac{MQ}{QP} \quad ---- [\text{By property of angle bisector of a triangle}]
\]
\[ MN = \frac{2}{3.5} = \frac{20}{35} = \frac{4}{7} \]
\[ MN : NP = 4 : 7 \]

4. In the adjoining figure, ray YM is the bisector of \( \angle XYZ \), where \( XY \equiv YZ \). Find the relation between XM and MZ. [2 marks]

**Solution:**

In \( \triangle XYZ \),
Ray YM is the angle bisector of \( \angle XYZ \) \[ \text{[Given]} \]
\[ \frac{XM}{MZ} = \frac{XY}{YZ} \] \[ \text{[By property of angle bisector of a triangle]} \]
seg \( XY \equiv \) seg \( YZ \) \[ \text{[Given]} \]
\[ XY = YZ \]
\[ \frac{XY}{YZ} = 1 \] \[ \text{(ii)} \]
\[ \frac{XM}{MZ} = 1 \] \[ \text{[From (i) and (ii)]} \]
\[ XM = MZ \]
\[ \text{seg } XM \equiv \text{seg } MZ \]

5. In the adjoining figure, ray PT is the bisector of \( \angle QPR \). Find the value of \( x \) and the perimeter of \( \triangle PQR \). [Mar 14] [3 marks]

**Solution:**

In \( \triangle PQR \),
Ray PT is the angle bisector of \( \angle QPR \).
\[ \frac{PQ}{PR} = \frac{QT}{TR} \] \[ \text{[By property of angle bisector of a triangle]} \]
\[ 5.6 \times x = 4 \times 5 \]
\[ 5.6 \times x = 20 \]
\[ \frac{20}{5.6} = x \]
\[ x = 7 \text{ cm} \]
\[ PR = 7 \text{ cm} \] \[ \text{[\because PR = x]} \]
Now, \( QR = QT + TR \) \[ \text{[\because T-R-Q]} \]
\[ QR = 4 + 5 \]
\[ QR = 9 \text{ cm} \]
Perimeter of \( \triangle PQR = PQ + QR + PR \)
\[ = 5.6 + 9 + 7 = 21.6 \text{ cm} \]
\[ \therefore \] The value of \( x \) is 7 cm and the perimeter of \( \triangle PQR \) is 21.6 cm.

6. In the adjoining figure, if \( ML \parallel BC \) and \( NL \parallel DC \).

Then prove that \( \frac{AM}{AB} = \frac{AN}{AD} \). [3 marks]

**Proof:**

In \( \triangle ABC \),
seg \( ML \parallel \) side BC \[ \text{[Given]} \]
\[ \frac{AM}{MB} = \frac{AL}{LC} \] \[ \text{[By B.P.T.]} \]
In \( \triangle ADC \),
seg \( NL \parallel \) side DC \[ \text{[Given]} \]
\[ \frac{AN}{ND} = \frac{AL}{LC} \quad \text{---- (ii) [By B.P.T.]} \]
\[ \frac{AM}{MB} = \frac{AN}{ND} \quad \text{---- [From (i) and (ii)]} \]
\[ \frac{MB}{AN} = \frac{ND}{AM} \quad \text{---- [By invertendo]} \]
\[ \frac{MB + AM}{AM} = \frac{ND + AN}{AN} \quad \text{---- [By componendo]} \]
\[ \frac{AB}{AM} = \frac{AD}{AN} \quad \text{---- [A–M–B, A–N–D]} \]
\[ \frac{AM}{AB} = \frac{AN}{AD} \quad \text{---- [By invertendo]} \]

7. As shown in the adjoining figure, in \( \triangle PQR \), seg PM is the median. Bisectors of \( \angle PMQ \) and \( \angle PMR \) intersect side PQ and side PR in points X and Y respectively, then prove that XY \( \parallel \) QR. [3 marks]

**Proof:**

Draw line XY.

In \( \triangle PMQ \),
ray MX is the angle bisector of \( \angle PMQ \). \[ \text{---- [Given]} \]
\[ \frac{MP}{MQ} = \frac{PX}{QX} \quad \text{---- (i) [By property of angle bisector of a triangle]} \]

In \( \triangle PMR \),
ray MY is the angle bisector of \( \angle PMR \). \[ \text{---- [Given]} \]
\[ \frac{MP}{MR} = \frac{PY}{RY} \quad \text{---- (ii) [By property of angle bisector of a triangle]} \]

But, seg PM is the median \[ \text{---- [Given]} \]
\[ M \text{ is midpoint of seg QR.} \]
\[ MQ = MR \quad \text{---- (iii)} \]
\[ \frac{PX}{QX} = \frac{PY}{RY} \quad \text{---- [From (i), (ii) and (iii)]} \]

In \( \triangle PQR \), seg XY \( \parallel \) seg QR \[ \text{---- [By converse of B.P.T.]} \]

8. \( \square ABCD \) is a trapezium in which AB \( \parallel \) DC and its diagonals intersect each other at the point O. Show that \( \frac{AO}{BO} = \frac{CO}{DO}. \) [3 marks]

**Proof:**

\( \square ABCD \) is a trapezium.
side AB \( \parallel \) side DC and seg AC is a transversal.
\[ \angle BAC \cong \angle DCA \quad \text{---- (i) [Alternate angles]} \]
In \( \triangle AOB \) and \( \triangle COD \),
\[ \angle BAO \cong \angle DCO \quad \text{---- [From (i) and A–O–C]} \]
\[ \angle AOB \cong \angle COD \quad \text{---- [Vertically opposite angles]} \]
\[ \triangle AOB \sim \triangle COD \quad \text{---- [By A–A test of similarity]} \]
\[ \frac{AO}{CO} = \frac{BO}{DO} \quad \text{---- [c.s.s.t.]} \]
\[ \frac{AO}{BO} = \frac{CO}{DO} \quad \text{---- [By alternendo]} \]
9. In the adjoining figure, \( \square ABCD \) is a trapezium. Side \( AB \parallel \text{seg} \ PQ \parallel \text{side} \ DC \) and \( AP = 15, PD = 12, QC = 14 \), then find \( BQ \). [2 marks]

**Solution:**

Side \( AB \parallel \text{seg} \ PQ \parallel \text{side} \ DC \) ---- [Given]

\[
\frac{AP}{PD} = \frac{BQ}{QC} \quad ---- \text{[By property of intercepts made by three parallel lines on a transversal]}
\]

\[
\frac{15}{12} = \frac{BQ}{14} \quad ---- \text{[\( \therefore \) AP = 15, PD = 12 and QC = 14]}
\]

\[
BQ = \frac{15 \times 14}{12}
\]

\[
\therefore \ BQ = 17.5
\]

10. Using the converse of Basic Proportionality Theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side and is half of it. [4 marks]

**Given:** In \( \triangle ABC \), \( P \) and \( Q \) are midpoints of sides \( AB \) and \( AC \) respectively.

**To Prove:** \( \text{seg} \ PQ \parallel \text{side} \ BC \)

\( PQ = \frac{1}{2} BC \)

**Proof:**

\( AP = PB \quad ---- \text{[\( P \) is the midpoint of side \( AB \).]} \)

\[
\frac{AP}{PB} = 1 \quad ---- \text{(i)}
\]

\( AQ = QC \quad ---- \text{[\( Q \) is the midpoint of side \( AC \).]} \)

\[
\frac{AQ}{QC} = 1 \quad ---- \text{(ii)}
\]

In \( \triangle ABC \),

\[
\frac{AP}{PB} = \frac{AQ}{QC} \quad ---- \text{[From (i) and (ii)]}
\]

\[
\therefore \ \text{seg} \ PQ \parallel \text{side} \ BC \quad ---- \text{(iii) [By converse of B.P.T.]}\]

In \( \triangle ABC \) and \( \triangle APQ \),

\( \angle ABC \cong \angle APQ \quad ---- \text{[From (iii), corresponding angles]} \)

\( \angle BAC \cong \angle PAQ \quad ---- \text{[Common angle]} \)

\[
\triangle ABC \sim \triangle APQ \quad ---- \text{[By A–A test of similarity]}\]

\[
\frac{AB}{AP} = \frac{BC}{PQ} \quad ---- \text{[c.s.s.t.]}\]

\[
\frac{AP + PB}{AP} = \frac{BC}{PQ} \quad ---- \text{[A–P–B]}\]

\[
\frac{AP + AP}{AP} = \frac{BC}{PQ} \quad ---- \text{[\( \therefore \) AP = PB]}\]

\[
\frac{2AP}{AP} = \frac{BC}{PQ}\]

\[
\frac{2}{1} = \frac{BC}{PQ}\]

\[
\therefore \ PQ = \frac{1}{2} BC\]
### 1.3 Similarity

Two figures are called similar if they have the same shapes not necessarily the same size.

**Properties of Similar Triangles:**

1. **Reflexivity:** \( \triangle ABC \sim \triangle ABC \). It means a triangle is similar to itself.
2. **Symmetry:** If \( \triangle ABC \sim \triangle DEF \), then \( \triangle DEF \sim \triangle ABC \).
3. **Transitivity:** If \( \triangle ABC \sim \triangle DEF \) and \( \triangle DEF \sim \triangle PQR \), then \( \triangle PQR \sim \triangle ABC \).

#### Exercise 1.3

1. Study the following figures and find out in each case whether the triangles are similar. Give reason.

   ![diagram](image.png)

   **Solution:**

   i. \( \triangle MTP \) and \( \triangle MNK \) are similar.
      
      **Reason:**
      
      - \( MN = MT + TN \)  
        \( = 2 + 4 = 6 \) units  
        \( \therefore \) \( MN = \frac{2}{6} = \frac{1}{3} \)  
      - \( MK = MP + PK \)  
        \( = 3 + 6 = 9 \) units  
        \( \therefore \) \( MK = \frac{3}{9} = \frac{1}{3} \)  
      - In \( \triangle MTP \) and \( \triangle MNK \),  
        \( \frac{MT}{MN} = \frac{MP}{MK} \)  
        \( \angle TMP \equiv \angle NMK \)  
        \( \therefore \) \( \triangle MTP \sim \triangle MNK \)  

   ii. \( \triangle PRT \) and \( \triangle PXS \) are not similar.
      
      **Reason:**
      
      - \( PX = PR + RX \)  
        \( = a + 2a = 3a \)  
        \( \therefore \) \( PX = \frac{a}{3a} = \frac{1}{3} \)  
      - \( RT = \frac{2b}{3b} = \frac{2}{3} \)  
        \( \therefore \) \( RT \neq \frac{RT}{XS} \)  
      - The corresponding sides of the two triangles are not in proportion.  
        \( \therefore \) \( \triangle PRT \) and \( \triangle PXS \) are not similar.
iii. \( \triangle DMN \) and \( \triangle AQR \) are similar.

**Reason:**

In \( \triangle DMN \) and \( \triangle AQR \),

\[
\angle DMN \cong \angle AQR \quad ---- \text{[Each is 55°]}
\]

\[
\angle DNM \cong \angle ARQ \quad ---- \text{[Each is of same measure]}
\]

\[\therefore \triangle DMN \sim \triangle AQR \quad ---- \text{[By A-A test of similarity]}\]

2. In the adjoining figure, \( \triangle ABC \) is right angled at \( B \).

D is any point on \( AB \). seg DE \( \perp \) seg AC.

If \( AD = 6 \text{ cm}, AB = 12 \text{ cm}, AC = 18 \text{ cm} \). Find \( AE \). \[2 \text{ marks}\]

**Solution:**

In \( \triangle AED \) and \( \triangle ABC \),

\[
\angle AED \cong \angle ABC \quad ---- \text{[Each is 90°]}
\]

\[
\angle DAE \cong \angle BAC \quad ---- \text{[Common angle]}
\]

\[\therefore \triangle AED \sim \triangle ABC \quad ---- \text{[By A-A test of similarity]}\]

\[\frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC} \quad ---- \text{[c.s.s.t.]}\]

\[\Rightarrow \frac{AE}{AB} = \frac{AD}{AC}\]

\[\Rightarrow \frac{AE}{12} = \frac{6}{18}\]

\[\Rightarrow AE = \frac{6 \times 12}{18} \]

\[\Rightarrow AE = 4 \text{ cm}\]

3. In the adjoining figure, \( E \) is a point on side \( CB \) produced of an isosceles \( \triangle ABC \) with \( AB = AC \). If \( AD \perp BC \) and \( EF \perp AC \), prove that \( \triangle ABD \sim \triangle ECF \). \[3 \text{ marks}\]

**Proof:**

In \( \triangle ABC \),

seg \( AB \cong \) seg \( AC \quad ---- \text{[Given]}\)

\( \angle B \cong \angle C \quad ---- \text{(i) [By isosceles triangle theorem]}\)

In \( \triangle ABD \) and \( \triangle ECF \),

\( \angle ABD \cong \angle ECF \quad ---- \text{[From (i)]}\)

\( \angle ADB \cong \angle EFC \quad ---- \text{[Each is 90°]}\)

\[\therefore \triangle ABD \sim \triangle ECF \quad ---- \text{[By A-A test of similarity]}\]

4. \( D \) is a point on side \( BC \) of \( \triangle ABC \) such that \( \angle ADC = \angle BAC \). Show that \( AC^2 = BC \times DC \). \[3 \text{ marks}\]

**Proof:**

In \( \triangle ACB \) and \( \triangle DCA \),

\( \angle BAC \cong \angle ADC \quad ---- \text{[Given]}\)

\( \angle ACB \cong \angle DCA \quad ---- \text{[Common angle]}\)

\[\therefore \triangle ACB \sim \triangle DCA \quad ---- \text{[By A-A test of similarity]}\]

\[\therefore \frac{AC}{DC} = \frac{BC}{AC} \quad ---- \text{[c.s.s.t.]}\]

\[\therefore \frac{AC}{DC} = \frac{BC}{AC}\]

\[\therefore \frac{AC^2}{DC} = BC \times DC\]
5. A vertical pole of length 6 m casts a shadow of 4 m long on the ground. At the same time, a tower casts a shadow 28 m long. Find the height of the tower.

Solution:
- AB represents the length of the pole.
  \[ \therefore AB = 6 \text{ m} \]
- BC represents the shadow of the pole.
  \[ \therefore BC = 4 \text{ m} \]
- PQ represents the height of the tower.
- QR represents the shadow of the tower.
  \[ \therefore QR = 28 \text{ m} \]

\[ \triangle ABC \sim \triangle PQR \]

\[ \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \]

\[ \because \text{vertical pole and tower are similar figures} \]

\[ \begin{align*}
\frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AC}{PR} \\
\therefore \frac{6}{PQ} &= \frac{1}{7} \\
\therefore 6 \times 7 &= PQ
\end{align*} \]

\[ \therefore PQ = 42 \text{ m} \]

\[ \therefore \text{Height of the tower is 42 m.} \]

6. Triangle ABC has sides of length 5, 6 and 7 units while \( \triangle PQR \) has perimeter of 360 units. If \( \triangle ABC \) is similar to \( \triangle PQR \), then find the sides of \( \triangle PQR \).

Solution:
- Since, \( \triangle ABC \sim \triangle PQR \)

\[ \begin{align*}
\frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AC}{PR} \\
\therefore \frac{5}{PQ} &= \frac{6}{QR} = \frac{7}{PR}
\end{align*} \]

By theorem on equal ratios,

\[ \begin{align*}
each \text{ ratio} &= \frac{5+6+7}{PQ+QR+PR} \\
&= \frac{18}{360} \\
&= \frac{1}{20}
\end{align*} \]

\[ \begin{align*}
\therefore \frac{5}{PQ} &= \frac{6}{QR} = \frac{7}{PR} = \frac{1}{20} \quad \text{(i)}
\end{align*} \]

\[ \begin{align*}
\frac{5}{PQ} &= \frac{1}{20} \\
\therefore PQ &= 20 \times 5 \\
\therefore PQ &= 100 \text{ units}
\end{align*} \]

\[ \begin{align*}
\frac{6}{QR} &= \frac{1}{20} \\
\therefore QR &= 20 \times 6 \\
\therefore QR &= 120 \text{ units}
\end{align*} \]

\[ \begin{align*}
\frac{7}{PR} &= \frac{1}{20} \\
\therefore PR &= 20 \times 7 \\
\therefore PR &= 140 \text{ units}
\end{align*} \]

\[ \therefore \triangle PQR \text{ has sides PQ, QR and PR of length 100 units, 120 units and 140 units respectively.} \]
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iii. \[
\frac{A(\triangle PBC)}{A(\triangle PQA)} = \frac{25}{1} \]

--- [By invertendo]

\[
\therefore \frac{A(\triangle PBC) - A(\triangle PQA)}{A(\triangle PQA)} = \frac{25 - 1}{1} \]

--- [By dividendo]

\[
\therefore \frac{A(\triangle QBCA)}{A(\triangle PQA)} = \frac{24}{1} \]

--- [By invertendo]

\[
\therefore \frac{A(\triangle PQA)}{A(\triangle QBCA)} = \frac{1}{24} \]

--- [By invertendo]

\[
\therefore \frac{A(\triangle PQA)}{A(\triangle QBCA)} = 1 : 24 \]

7. In the adjoining figure, DE \parallel BC and AD : DB = 5 : 4.

Find:  
   i. DE : BC  
   ii. DO : DC  
   iii. \(A(\triangle DOE) : A(\triangle DCE)\) 

[5 marks]

Solution:

i. DE \parallel BC  
   AB is a transversal

\[
\therefore \angle ADE \cong \angle ABC \]

--- (i) [Corresponding angles]

In \(\triangle ADE\) and \(\triangle ABC\),

\[
\angle ADE \cong \angle ABC \]

--- [From (i)]

\[
\angle DAE \cong \angle BAC \]

--- [Common angle]

\[
\therefore \triangle ADE \sim \triangle ABC \]

--- [By \(A-A\) test of similarity]

\[
\frac{AD}{AB} = \frac{DE}{BC} \]

--- (ii) [c.s.s.t.]

[Substituting the given values]

\[
\frac{AD}{AB} = \frac{5}{4} \]

--- [By invertendo]

\[
\frac{DB + AD}{AD} = \frac{4 + 5}{5} \]

--- [By componendo]

\[
\frac{AB}{AD} = \frac{9}{5} \]

--- [A–D–B]

\[
\frac{AD}{AB} = \frac{5}{9} \]

--- (iii) [By invertendo]

\[
\frac{DE}{BC} = \frac{5}{9} \]

--- (iv) [From (ii) and (iii)]

\[
\therefore \text{DE : BC} = 5 : 9 \]

ii. In \(\triangle DOE\) and \(\triangle COB\),

\[
\angle EDO \cong \angle BCO \]

--- [Alternate angles on parallel lines DE and BC]

\[
\angle DOE \cong \angle COB \]

--- [Vertically opposite angles]

\[
\therefore \triangle DOE \sim \triangle COB \]

--- [By \(A-A\) test of similarity]

\[
\frac{DO}{OC} = \frac{DE}{BC} \]

--- [c.s.s.t.]

[From (iv)]

\[
\frac{DO}{OC} = \frac{5}{9} \]

--- [By invertendo]

\[
\frac{OC}{DO} = \frac{9}{5} \]

--- [By componendo]

\[
\frac{OC + DO}{DO} = \frac{9 + 5}{5} \]

--- [By invertendo]

\[
\therefore \text{DO : DC} = 5 : 14 \]
iii. \[\frac{A(\triangle DOE)}{A(\triangle DCE)} = \frac{DO}{DC}\]  
\[\text{[Ratio of areas of two triangles having equal heights is equal to the ratio of the corresponding bases]}\]

\[\therefore \frac{A(\triangle DOE)}{A(\triangle DCE)} = \frac{5}{14}\]  
\[\text{[From (v)]}\]

\[\therefore A(\triangle DOE) : A(\triangle DCE) = 5 : 14\]

8. In the adjoining figure, seg AB || seg DC.

Using the information given, find the value of \(x\).  
[3 marks]

Solution:

\[
\angle ABD \cong \angle CDB \quad \text{[Alternate angles]}\]

\[
\angle ABO \cong \angle CDO \quad \text{[From (i), D - O - B]}\]

\[
\angle AOB \cong \angle COD \quad \text{[Vertically opposite angles]}\]

\[
\triangle AOB \sim \triangle COD \quad \text{[By A-A test of similarity]}\]

\[
\frac{OA}{OC} = \frac{OB}{OD} \quad \text{[c.s.s.t]}\]

\[
\frac{3x - 19}{x - 5} = \frac{x - 3}{3} \quad \text{[Substituting the given values]}\]

\[
3(3x - 19) = (x - 3)(x - 5)\]

\[
9x - 57 = x^2 - 8x + 15\]

\[
x^2 - 17x + 72 = 0\]

\[
(x - 9)(x - 8) = 0\]

\[
x - 9 = 0 \text{ or } x - 8 = 0\]

\[
x = 9 \text{ or } x = 8\]

9. Using the information given in the adjoining figure, find \(\angle F\).  
[3 marks]

Solution:

\[
\frac{AB}{DE} = \frac{3.8}{7.6} = \frac{1}{2}\]  
\[\text{[Substituting the given values]}\]

\[
\frac{BC}{EF} = \frac{6}{12} = \frac{1}{2}\]  
\[\text{[Substituting the given values]}\]

\[
\frac{CA}{FD} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}\]  
\[\text{[Substituting the given values]}\]

In \(\triangle ABC\) and \(\triangle DEF\),

\[
\angle AOB \cong \angle CDF \quad \text{[By S-S-S test of similarity]}\]

\[
\angle C \cong \angle F \quad \text{[c.a.s.t]}\]

In \(\triangle ABC\),

\[
\angle A + \angle B + \angle C = 180^\circ\]  
\[\text{[Sum of the measures of all angles of a triangle is } 180^\circ.]\]

\[
80^\circ + 60^\circ + \angle C = 180^\circ\]  
\[\text{[Substituting the given values]}\]

\[
\angle C = 180^\circ - 140^\circ\]

\[
\angle C = 40^\circ\]  
\[\text{[v]}\]

\[
\angle F = 40^\circ\]  
\[\text{[From (iv) and (v)]}\]
10. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow of length 40 m on the ground. Determine the height of the tower.

Solution:
Let AB represent the vertical stick, AB = 12 m.
BC represents the shadow of the stick, BC = 8 m.
PQ represents the height of the tower.
QR represents the shadow of the tower, QR = 40 m.

\[ \Delta ABC \sim \Delta PQR \]

\[ \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \]

\[ \frac{12}{PQ} = \frac{8}{40} \]

\[ PQ = 12 \times 5 = 60 \]

\[ \text{The height of the tower is 60 m.} \]

11. In each of the figures, an altitude is drawn to the hypotenuse. The lengths of different segments are marked in each figure. Determine the value of \( x, y, z \) in each case.

Solution:

i. In \( \Delta ABC \), \( \angle ABC = 90^\circ \)

seg BD \( \perp \) hypotenuse AC

\[ BD^2 = AD \times DC \]

\[ y^2 = 4 \times 5 \]

\[ y = \sqrt{4 \times 5} \]

\[ y = 2\sqrt{5} \]

\[ x = 6 \]

ii. In \( \Delta ADB \),

\[ \angle ADB = 90^\circ \]

\[ AB^2 = AD^2 + BD^2 \]

\[ x^2 = (4)^2 + y^2 \]

\[ x^2 = 4^2 + (2\sqrt{5})^2 \]

\[ x^2 = 16 + 20 \]

\[ x^2 = 36 \]

\[ x = 6 \]

iii. In \( \Delta BDC \),

\[ \angle BDC = 90^\circ \]

\[ BC^2 = BD^2 + CD^2 \]

\[ z^2 = y^2 + (5)^2 \]

\[ z^2 = (2\sqrt{5})^2 + (5)^2 \]

\[ z^2 = 20 + 25 \]

\[ z^2 = 45 \]

\[ z = \sqrt{5\times 5} \]

\[ z = 3\sqrt{5} \]

\[ x = 6, y = 2\sqrt{5} \text{ and } z = 3\sqrt{5} \]
ii. In $\triangle PSQ$,

\[ m \angle PSQ = 90^\circ \quad \Rightarrow \quad \text{[Seg QS } \perp \text{ hypotenuse PR]} \]

\[ \therefore \quad PQ^2 = PS^2 + QS^2 \quad \Rightarrow \quad \text{[By Pythagoras theorem]} \]

\[ (6)^2 = (4)^2 + y^2 \quad \Rightarrow \quad \text{[Substituting the given values]} \]

\[ 36 = 16 + y^2 \]

\[ y^2 = 20 \]

\[ \therefore \quad y = \sqrt{4 \times 5} \quad \Rightarrow \quad \text{[Taking square root on both sides]} \]

\[ y = 2 \sqrt{5} \quad \Rightarrow \quad (i) \]

In $\triangle PQR$,

\[ \text{seg QS } \perp \text{ hypotenuse PR} \quad \Rightarrow \quad \text{[Given]} \]

\[ QS^2 = PS \times SR \quad \Rightarrow \quad \text{[By the property of geometric mean]} \]

\[ y^2 = 4 \times x \quad \Rightarrow \quad \text{[Substituting the given values]} \]

\[ (2 \sqrt{5})^2 = 4x \quad \Rightarrow \quad \text{[From (i)]} \]

\[ 20 = 4x \]

\[ x = \frac{20}{4} \]

\[ x = 5 \quad \Rightarrow \quad (ii) \]

In $\triangle QSR$,

\[ m \angle QSR = 90^\circ \quad \Rightarrow \quad \text{[Seg QS } \perp \text{ hypotenuse PR]} \]

\[ QR^2 = QS^2 + SR^2 \quad \Rightarrow \quad \text{[By Pythagoras theorem]} \]

\[ z^2 = y^2 + x^2 \quad \Rightarrow \quad \text{[Substituting the given values]} \]

\[ z^2 = (2 \sqrt{5})^2 + (5)^2 \quad \Rightarrow \quad \text{[From (i) and (ii)]} \]

\[ z^2 = 20 + 25 \]

\[ z^2 = 45 \]

\[ z = \sqrt{5 \times 5} \quad \Rightarrow \quad \text{[Taking square root on both sides]} \]

\[ z = 3 \sqrt{5} \]

\[ \therefore \quad x = 5, y = 2 \sqrt{5} \text{ and } z = 3 \sqrt{5} \]

12. $\triangle ABC$ is a right angled triangle with $\angle A = 90^\circ$. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle. [4 marks]

Construction: Let P, Q and R be the points of contact of tangents AC, AB and BC respectively and draw segments OP and OQ.

Solution:

In $\triangle ABC$,

\[ \angle BAC = 90^\circ \quad \Rightarrow \quad \text{[Given]} \]

\[ \therefore \quad BC^2 = AC^2 + AB^2 \quad \Rightarrow \quad \text{[By Pythagoras theorem]} \]

\[ BC^2 = (6)^2 + (8)^2 \quad \Rightarrow \quad \text{[Substituting the given values]} \]

\[ BC^2 = 36 + 64 \]

\[ BC^2 = 100 \]

\[ BC = 10 \text{ units} \quad \Rightarrow \quad \text{(i) [Taking square root on both sides]} \]

Let the radius of the circle be $x$ cm.

\[ \therefore \quad OP = OQ = x \quad \Rightarrow \quad \text{[Radii of same circle]} \]

In $\square OPQ$,

\[ \angle OPA = \angle OQA = 90^\circ \quad \Rightarrow \quad \text{[Radius is } \perp \text{ to the tangent]} \]

\[ \angle PAQ = 90^\circ \quad \Rightarrow \quad \text{[Given]} \]

\[ \angle POQ = 90^\circ \quad \Rightarrow \quad \text{[Remaining angle]} \]

\[ \therefore \quad \square OPQ \text{ is a rectangle} \quad \Rightarrow \quad \text{[By definition]} \]

But, $OP = OQ$ \quad \Rightarrow \quad [Radii of same circle]

\[ \therefore \quad \square OPQ \text{ is a square} \quad \Rightarrow \quad [A \text{ rectangle is a square if its adjacent sides are congruent}] \]

\[ OP = OQ = QA = AP = x \quad \Rightarrow \quad [Sides of a square] \]
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Now, $AQ + BQ = AB$ ---- [A–Q–B]
\[x + BQ = 8\] ---- [Substituting the given values]
\[BQ = 8 - x\]
\[AP + CP = AC\] ---- [A–P–C]
\[x + CP = 6\] ---- [Substituting the given values]
\[CP = 6 - x\]
\[BQ = BR = 8 - x\] ---- (ii) [Length of tangent segments drawn from a external point to the circle are equal.]
\[CP = CR = 6 - x\] ---- (iii)
\[BC = CR + BR\] ---- (iv) [C–R–B]
\[10 = 6 - x + 8 - x\] ---- [From (i), (ii), (iii) and (iv)]
\[2x = 4\]
\[x = 2\]
\[\therefore\] The radius of the circle is 2 cm.

13. In $\Delta PQR$, seg PM is a median. If $PM = 9$ and $PQ^2 + PR^2 = 290$, find $QR$. [2 marks]

Solution:

In $\Delta PQR$, seg PM is the median ---- [Given]
\[PQ^2 + PR^2 = 2PM^2 + 2MR^2\] ---- [By Apollonius theorem]
\[290 = 2(9)^2 + 2MR^2\] ---- [Substituting the given values]
\[290 = 2(81) + 2MR^2\]
\[290 = 162 + 2MR^2\]
\[2MR^2 = 290 - 162\]
\[2MR^2 = 128\]
\[MR^2 = \frac{128}{2}\]
\[MR^2 = 64\]
\[MR = 8\] ---- (i) [Taking square root on both sides]
Also, $QR = 2MR$ ---- [∴ M is the midpoint of seg QR]
\[QR = 2 \times 8\] ---- [From (i)]
\[\therefore QR = 16\]

14. From the information given in the adjoining figure, prove that: $PM = PN = \sqrt{3} \times a$, where $QR = a$. [4 marks]

Proof:

In $\Delta PMR$,
\[QM = QR = a\] ---- [Given]
\[Q\] is midpoint of seg MR.
\[\therefore\] seg PQ is the median
\[PM^2 + PR^2 = 2PQ^2 + 2QM^2\] ---- [By Apollonius theorem]
\[PM^2 + a^2 = 2a^2 + 2a^2\] ---- [Substituting the given values]
\[PM^2 + a^2 = 4a^2\]
\[PM^2 = 3a^2\]
\[\therefore PM = \sqrt{3}a\] ---- [Taking square root on both sides]
Similarly, we can prove
\[PN = \sqrt{3}a\]
\[\therefore PM = PN = \sqrt{3}a\]
15. D and E are the points on sides AB and AC such that \(AB = 5.6, AD = 1.4, AC = 7.2\) and \(AE = 1.8\). Show that \(DE \parallel BC\). \([2\text{ marks}]\)

**Proof:**

\[
\begin{align*}
\triangle ABD & \sim \triangle AEC \\
\frac{DB}{AB} & = \frac{AD}{AC} \\
\therefore \frac{DB}{5.6} & = \frac{1.4}{7.2} \\
\therefore DB & = \frac{1.4 \times 5.6}{7.2} \\
& = \frac{7.84}{7.2} \\
& = 1.083333333 \\
\end{align*}
\]

Also, \(EC = AC - AE\)

\[
\begin{align*}
\frac{EC}{AC} & = \frac{7.2 - 1.8}{7.2} \\
& = \frac{5.4}{7.2} \\
& = \frac{1}{3} \\
\end{align*}
\]

In \(\triangle ABC\),

\[
\frac{AD}{DB} = \frac{AE}{EC} \\
\frac{1.4}{4.2} = \frac{1.8}{5.4} \\
\]

\(\therefore seg DE \parallel seg BC\) \([By\ converse\ of\ B.P.T.]\)

16. In \(\triangle PQR\), if QS is the angle bisector of \(\angle Q\), then show that \(\frac{A(\triangle PQS)}{A(\triangle QRS)} = \frac{PQ}{QR}\). \([3\text{ marks}]\)

(Hint: Draw QT \(\perp PR\))

**Proof:**

In \(\triangle PQR\),

Ray QS is the angle bisector of \(\angle PQR\) \([Given]\)

\[
\frac{PQ}{QR} = \frac{PS}{SR} \\
\]

Height of \(\triangle PQS = \) Height of \(\triangle QRS = QT\)

\[
\frac{A(\triangle PQS)}{A(\triangle QRS)} = \frac{PS}{SR} \\
\]

\[
\frac{A(\triangle PQS)}{A(\triangle QRS)} = \frac{PQ}{QR} \\
\]

\(\therefore seg DE \parallel seg BC\) \([By\ converse\ of\ B.P.T.]\)

17. In the adjoining figure, \(XY \parallel AC\) and \(XY\) divides the triangular region \(ABC\) into two equal areas. Determine \(AX : AB\). \([4\text{ marks}]\)

**Solution:**

seg \(XY \parallel\) side AC on transversal BC

\(\angle XYB \cong \angle ACB\) \([i]\)

In \(\triangle XYB\) and \(\triangle ACB\),

\(\angle XYB \cong \angle ACB\) \([From\ (i)]\)

\(\angle ABC \cong \angle YXB\) \([Common\ angle]\)

\(\triangle XYB \sim \triangle ACB\) \([By\ A-A\ test\ of\ similarity]\)

\[
\frac{A(\triangle XYB)}{A(\triangle ACB)} = \frac{XB^2}{AB^2} \\
\]

Now, \(A(\triangle XYB) = \frac{1}{2} A(\triangle ACB)\) \([\because seg XY\ divides\ the\ triangular\ region\ ABC\ into\ two\ equal\ areas]\)

\[
\frac{A(\triangle XYB)}{A(\triangle ACB)} = \frac{1}{2} \\
\]

\(\therefore XB = \frac{1}{\sqrt{2}} AB\) \([From\ (ii)\ and\ (iii)]\)

\[\therefore XA = \frac{1}{\sqrt{2}} AB\] \([Taking\ square\ root\ on\ both\ sides]\)
18. Let X be any point on side BC of $\triangle ABC$, XM and XN are drawn parallel to BA and CA. MN meets produced BC in T. Prove that $TX^2 = TB \cdot TC$. [4 marks]

**Proof:**

- In $\triangle TXM$,
  
  $\frac{TN}{NM} = \frac{TB}{BX}$ ---- [Given]

- In $\triangle TMC$,
  
  $\frac{TN}{NM} = \frac{TX}{CX}$ ---- [Given]

- From (i) and (ii)
  
  $\frac{TX}{TB} = \frac{TC}{TX}$ ---- [By invertendo]

- $TX^2 = TB \cdot TC$

19. Two triangles, $\triangle ABC$ and $\triangle DBC$, lie on the same side of the base BC. From a point P on BC, PQ || AB and PR || BD are drawn. They intersect AC at Q and DC at R. Prove that QR || AD. [3 marks]

**Proof:**

- In $\triangle CAB$,
  
  $\frac{CP}{PB} = \frac{CQ}{AQ}$ ---- [Given]

- In $\triangle BCD$,
  
  $\frac{CP}{PB} = \frac{CR}{RD}$ ---- [Given]

- From (i) and (ii)
  
  $\frac{CQ}{AQ} = \frac{CR}{RD}$ ---- [By converse of B.P.T.]
20. In the figure, \( \triangle ADB \) and \( \triangle CDB \) are on the same base \( DB \).
If \( AC \) and \( BD \) intersect at \( O \), then prove that
\[
\frac{A(\triangle ADB)}{A(\triangle CDB)} = \frac{AO}{CO} \quad [3 \text{ marks}]
\]

**Proof:**
\[
\frac{A(\triangle ADB)}{A(\triangle CDB)} = \frac{AN}{CM} \\
\text{[Ratio of areas of two triangles with the same} \\
\text{base is equal to the ratio of their corresponding]}
\]
In \( \triangle ANO \) and \( \triangle CMO \),
\[
\angle ANO \cong \angle CMO \quad \text{[Each is 90°]} \\
\angle AON \cong \angle COM \quad \text{[Vertically opposite angles]}
\]
\[
\therefore \triangle ANO \sim \triangle CMO \quad \text{[By A-A test of similarity]}
\]
\[
\frac{AN}{CM} = \frac{AO}{CO} \quad (ii) \quad \text{[c.s.s.t.]} \]
\[
\therefore \frac{A(\triangle ADB)}{A(\triangle CDB)} = \frac{AO}{CO} \quad \text{[From (i) and (ii)]}
\]

21. In \( \triangle ABC \), \( D \) is a point on \( BC \) such that \( \frac{BD}{DC} = \frac{AB}{AC} \). Prove that \( AD \) is the bisector of \( \angle A \).
(Hint: Produce \( BA \) to \( E \) such that \( AE = AC \). Join \( EC \)) \quad [5 \text{ marks}]

**Proof:**
\[
\text{seg } BA \text{ is produced to point } E \text{ such that } AE = AC \text{ and seg } EC \text{ is drawn.}
\]
\[
\frac{BD}{DC} = \frac{AB}{AC} \quad \text{[Given]} \\
\frac{AC}{AE} = \frac{BD}{DC} \quad \text{[By construction]} \\
\therefore \frac{BD}{DC} = \frac{AB}{AE} \quad \text{[Substituting (ii) in (i)]}
\]
\[
\therefore \text{seg } AD \parallel \text{ seg } EC \quad \text{[By converse of B.P.T.]} \]
On transversal \( BE \),
\[
\angle BAD \cong \angle BEC \quad \text{[Corresponding angles]} \]
\[
\therefore \angle BAD \cong \angle AEC \quad \text{[Alternate angles]} \]
On transversal \( AC \),
\[
\angle CAD \cong \angle ACE \quad \text{[Alternate angles]} \]
\[
\begin{align*}
\angle AEC & \cong \angle ACE \\
\therefore \angle BAD & \cong \angle CAD \\
\therefore \angle BAD & \cong \angle CAD \\
\therefore \text{Ray } AD \text{ is the bisector of } \angle BAC
\end{align*}
\]

22. The bisector of interior \( \angle A \) of \( \triangle ABC \) meets \( BC \) in \( D \). The
bisector of exterior \( \angle A \) meets \( BC \) produced in \( E \). Prove that
\[
\frac{BD}{BE} = \frac{CD}{CE} \quad \text{[Hint: For the bisector of } \angle A \text{ which is exterior of } \triangle ABC \text{,}\frac{AB}{AC} = \frac{BE}{CE}}
\]
\quad [5 \text{ marks}]

**Construction:** Draw \( \text{seg } CP \parallel \text{ seg } AE \) meeting \( AB \) at point \( P \).

**Proof:**
\[
\text{In } \triangle ABC, \\
\text{Ray } AD \text{ is bisector of } \angle BAC \quad \text{[Given]} \\
\therefore \frac{AB}{AC} = \frac{BD}{CD} \quad \text{[i] \text{[By property of angle bisector of triangle]}}
\]
In \( \triangle ABE \),
\[
\frac{\text{seg CP}}{\text{seg AE}} = \frac{\text{BP}}{\text{AP}}
\]

\[
\frac{\text{BC} + \text{CE}}{\text{CE}} = \frac{\text{BP} + \text{AP}}{\text{AP}}
\]

\[\therefore \frac{\text{BE}}{\text{CE}} = \frac{\text{AB}}{\text{AP}}\] (ii)
seg CP || seg AE on transversal BF.
\[
\angle FAE \cong \angle APC
\]

\[\therefore \triangle BCE \sim \triangle ACF\] (iii) [Corresponding angles]
seg CP || seg AE on transversal AC.
\[
\angle CAE \cong \angle ACP
\]

Also, \( \angle FAE \cong \angle CAE \) (v) [Seg AE bisects \( \angle FAC \)]
\[
\angle APC \cong \angle ACP
\]

In \( \triangle APC \),
\[
\angle APC \cong \angle ACP
\]

\[\therefore \triangle APC \cong \triangle ACP\] (vi) [From (iii), (iv) and (v)]
\[
\therefore \triangle ABC \sim \triangle ACF
\]

\[\therefore \angle AP = \angle AC\] (vii) [By converse of isosceles triangle theorem]

\[\therefore \frac{\text{BE}}{\text{CE}} = \frac{\text{AB}}{\text{AC}}\] (viii) [From (ii) and (vii)]

\[\therefore \frac{\text{BD}}{\text{CE}} = \frac{\text{BE}}{\text{AC}}\] (ix) [By alternendo]

**23.** In the adjoining figure, \( \square ABCD \) is a square. \( \triangle BCE \) on side BC and \( \triangle ACF \) on the diagonal AC are similar to each other. Then, show that \( \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{1}{2} \) [3 marks]

**Proof:**
\[\square ABCD \text{ is a square.}\] (i) [Given]
\[\therefore \text{AC} = \sqrt{2} \text{ BC}\] (i) \[\therefore \triangle BCE \sim \triangle ACF\] (i) [\( \therefore \text{Diagonal of a square} = \sqrt{2} \times \text{side of square}\)]
\[\therefore \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{(\text{BC})^2}{(\sqrt{2} \text{ BC})^2}\] (ii) [By theorem on areas of similar triangles]
\[\therefore \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{\text{BC}^2}{2 \text{ BC}^2}\]
\[\therefore \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{1}{2}\]
\[\therefore \frac{\text{Area}(\triangle BCE)}{\text{Area}(\triangle ACF)} = \frac{1}{2}\]

**24.** Two poles of height ‘a’ meters and ‘b’ metres are ‘p’ meters apart. Prove that the height ‘h’ drawn from the point of intersection N of the lines joining the top of each pole to the foot of the opposite pole is \( \frac{ab}{a+b} \) metres. [4 marks]

**Proof:**
\[\text{Let RT} = x \text{ and TQ} = y.\]
\[\text{In } \triangle PQR \text{ and } \triangle NTR,\]
\[\angle PQR \cong \angle NTR\] (i) [Each is 90°]
\[\angle PRQ \cong \angle NRT\] (ii) [Common angle]
\[ \therefore \triangle PQR \sim \triangle NTR \quad \text{[By A – A test of similarity]} \]
\[ \frac{PQ}{QR} = \frac{NT}{TR} \quad \text{[c.s.s.t.]} \]
\[ \frac{a}{h} = \frac{p}{x} \quad \text{[Substituting the given values]} \]
\[ x = \frac{ph}{a} \quad \text{[i]} \]

In \( \triangle SRQ \) and \( \triangle NTQ \),
\[ \angle SRQ \cong \angle NTQ \quad \text{[Each is 90°]} \]
\[ \angle SQR \cong \angle NQT \quad \text{[Common angle]} \]
\[ \triangle SRQ \sim \triangle NTQ \quad \text{[By A-A test of similarity]} \]
\[ \frac{SR}{NT} = \frac{QR}{QT} \quad \text{[c.s.s.t]} \]
\[ \frac{b}{h} = \frac{p}{y} \quad \text{[Substituting the given values]} \]
\[ y = \frac{ph}{b} \quad \text{[ii]} \]

\[ x + y = \frac{ph}{a} + \frac{ph}{b} \quad \text{[Adding (i) and (ii)]} \]
\[ p = ph \left( \frac{1}{a} + \frac{1}{b} \right) \quad \text{[R – T – Q]} \]
\[ \frac{p}{ph} = \frac{b + a}{ab} \]
\[ \frac{1}{h} = \frac{a + b}{ab} \]
\[ h = \frac{ab}{a + b} \text{ metres} \quad \text{[By invertendo]} \]

25. In the adjoining figure, \( \square DEFG \) is a square and \( \angle BAC = 90° \).
Prove that: i. \( \triangle AGF \sim \triangle DBG \)  
   ii. \( \triangle AGF \sim \triangle EFC \)  
   iii. \( \triangle DBG \sim \triangle EFC \)  
   iv. \( DE^2 = BD \cdot EC \)  

   [5 marks]

Proof:

i \( \square DEFG \) is a square. \quad [Given]
\[ \text{seg GF \parallel seg DE} \quad \text{[Opposite sides of a square]} \]
\[ \therefore \text{seg GF \parallel seg BC} \quad \text{(i) \[B-D-E-C\]} \]

In \( \triangle AGF \) and \( \triangle DBG \),
\[ \angle GAF \cong \angle BDF \quad \text{[Each is 90°]} \]
\[ \angle AGF \cong \angle DBG \quad \text{[Corresponding angles of parallel lines GF and BC]} \]
\[ \therefore \triangle AGF \sim \triangle DBG \quad \text{(ii) \[By A-A test of similarity\]} \]

ii In \( \triangle AGF \) and \( \triangle EFC \),
\[ \angle GAF \cong \angle FEC \quad \text{[Each is 90°]} \]
\[ \angle AFG \cong \angle ECF \quad \text{[Corresponding angles of parallel lines GF and BC]} \]
\[ \therefore \triangle AGF \sim \triangle EFC \quad \text{(iii) \[By A-A test of similarity\]} \]

iii. Since, \( \triangle AGF \sim \triangle DBG \) \quad \text{[From (ii)]}
and \( \triangle AGF \sim \triangle EFC \) \quad \text{[From (iii)]}
\[ \therefore \triangle DBG \sim \triangle EFC \quad \text{[From (ii) and (iii)]} \]
iv. Since, $\triangle DBG \sim \triangle EFC$

$$\frac{BD}{DG} = \frac{FE}{EC} \quad ---- [c.s.s.t.]$$

$$\therefore \quad DG \times FE = BD \times EC \quad ---- (iv)$$

But, $DG = EF = DE \quad ---- (v) \quad [\text{Sides of a square}]$

$$\therefore \quad DE \times DE = DB \times EC \quad ---- \text{[From (iv) and (v)]}$$

$$\therefore \quad DE^2 = BD \times EC$$

### One-Mark Questions

1. In $\triangle ABC$ and $\triangle XYZ$, \( \frac{AB}{YZ} = \frac{BC}{ZX} = \frac{AC}{XY} \), then state by which correspondence are $\triangle ABC$ and $\triangle XYZ$ similar.

**Solution:**

$\triangle ABC \sim \triangle XYZ$ by $ABC \leftrightarrow YZX$.

2. In the figure, $RP : PK = 3 : 2$.

Find $\frac{\triangle TRP}{\triangle TPK}$.

**Solution:**

Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.

$$\therefore \quad \frac{\triangle TRP}{\triangle TPK} = \frac{RP}{PK} = \frac{3}{2}$$

3. Write the statement of Basic Proportionality Theorem.

**Solution:**

If a line parallel to a side of a triangle intersects the other sides in two distinct points, then the line divides those sides in proportion.

4. What is the ratio among the length of the sides of any triangle of angles $30^\circ \rightarrow 60^\circ \rightarrow 90^\circ$?

**Solution:**

The ratio is $1 : \sqrt{3} : 2$.

5. What is the ratio among the length of the sides of any triangle of angles $45^\circ \rightarrow 45^\circ \rightarrow 90^\circ$?

**Solution:**

The ratio is $1 : 1 : \sqrt{2}$.

6. State the test by which the given triangles are similar.

**Solution:**

$\triangle ABC \sim \triangle EDC$ by SAS test of similarity.

7. In the adjoining figure, find $\frac{\triangle APQR}{\triangle ARSQ}$.

**Solution:**

Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.

$$\therefore \quad \frac{\triangle APQR}{\triangle ARSQ} = \frac{PQ}{ST}$$

8. Find the diagonal of a square whose side is 10 cm.

**Solution:**

Diagonal of a square $= \sqrt{2} \times$ side.

$$= \sqrt{2} \times (10) = 10 \sqrt{2} \text{ cm}$$

9. Adjacent sides of parallelogram are 11 cm and 17 cm respectively. If length of one diagonal is 26 cm, then using which theorem/property can we find the length of the other diagonal?

**Solution:**

We can find the length of the other diagonal by using Apollonius’ theorem.

10. In the adjoining figure, using given information, find $BC$.

**Solution:**

$$BC = \frac{\sqrt{3}}{2} \times AC \quad ---- \text{[Side opposite to } 60^\circ]$$

$$= \frac{\sqrt{3}}{2} \times 24$$

$$\therefore \quad BC = 12 \sqrt{3} \text{ units}$$
11. Find the value of MN, so that $A(\triangle ABC) = A(\triangle LMN)$.

**Solution:**

$A(\triangle ABC) = A(\triangle LMN)$

$\therefore \frac{1}{2} \times BC \times AD = \frac{1}{2} \times MN \times LP$

$\therefore \frac{1}{2} \times 5 \times 8 = \frac{1}{2} \times MN \times 4$

$\therefore MN = \frac{5 \times 8}{4}$

$\therefore MN = 10 \text{ cm}$

12. If the sides of a triangle are 6 cm, 8 cm and 10 cm respectively, determine whether the triangle is right angled triangle or not. [Mar 14]

**Solution:**

Note that, $6^2 + 8^2 = 10^2$,

$\therefore$ By converse of Pythagoras theorem, the given triangle is a right angled triangle.

13. Sides of the triangle are 7 cm, 24 cm and 25 cm. Determine whether the triangle is right-angled triangle or not. [Oct 14]

**Solution:**

The longest side is 25 cm.

$(25)^2 = 625$ ....(i)

Now, sum of the squares of the other two sides will be

$(7)^2 + (24)^2 = 49 + 576$

$= 625$ ....(ii)

$(25)^2 = (7)^2 + (24)^2$ ....[From (i) and (ii)]

Yes, the given sides form a right angled triangle.

....[By converse of Pythagoras theorem]

14. In the following figure

seg AB $\perp$ seg BC,

seg DC $\perp$ seg BC.

If $AB = 2$ and $DC = 3$,

find $\frac{A(\triangle ABC)}{A(\triangle DCB)}$. [Mar 15]

**Solution:**

Ratio of the areas of two triangles having equal base is equal to the ratio of their corresponding heights.

$A(\triangle ABC) = \frac{AB}{DC}$

$\therefore A(\triangle ABC) = \frac{2}{3}$

15. Find the diagonal of a square whose side is 16 cm. [July 15]

**Solution:**

Diagonal of a square $= \sqrt{2} \times$ side.

$= \sqrt{2} \times 16 = 16 \sqrt{2}$ cm

**Additional Problems for Practice**

**Based on Exercise 1.1**

1. In the adjoining figure, QR = 12 and SR = 4.

Find values of

i. $\frac{A(\triangle PSR)}{A(\triangle PQR)}$

ii. $\frac{A(\triangle PQS)}{A(\triangle PQR)}$

iii. $\frac{A(\triangle PQS)}{A(\triangle PSR)}$ [3 marks]

2. The ratio of the areas of two triangles with the equal heights is 3 : 4. Base of the smaller triangle is 15 cm. Find the corresponding base of the larger triangle. [2 marks]

3. In the adjoining figure, seg $AE \perp$ seg BC and seg $DF \perp$ seg BC.

Find

i. $\frac{A(\triangle ABC)}{A(\triangle DBC)}$

ii. $\frac{A(\triangle DBF)}{A(\triangle DFC)}$

iii. $\frac{A(\triangle AEC)}{A(\triangle DBF)}$ [2 marks]

**Based on Exercise 1.2**

4. In the adjoining figure, seg $EF \parallel$ side AC,

$AB = 18$, $AE = 10$,

$BF = 4$. Find BC. [3 marks]

5. In the adjoining figure, seg $DE \parallel$ side AC and seg DC $\parallel$ side AP.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$ [3 marks]
6. In the adjoining figure, PM = 10, MR = 8, QN = 5, NR = 4. State with reason whether line MN is parallel to side PQ or not. [2 marks]

7. In the following figure, in \( \triangle PQR \), seg RS is the bisector of \( \angle PRQ \), PS = 6, SQ = 8, PR = 15. Find QR. [Mar 15] [2 marks]

8. Bisectors of \( \angle B \) and \( \angle C \) in \( \triangle ABC \) meet each other at P. Line AP cuts the side BC at Q. Then prove that \( \frac{AP}{PQ} = \frac{AB + AC}{BC} \). [3 marks]

9. In the figure given below Ray LS is the bisector of \( \angle MLN \), where seg ML \( \cong \) seg LN, find the relation between MS and SN. [3 marks]

10. In the given figure, line \( l \parallel \) side BC, AP = 4, PB = 8, AY = 5 and YC = x. Find x. [July 15] [2 marks]

Based on Exercise 1.3

11. In the adjoining figure, \( \triangle MPL \sim \triangle NQL \), MP = 21, ML = 35, NQ = 18, QL = 24. Find PL and NL. [2 marks]

12. In the adjoining figure, \( \triangle PQR \) and \( \triangle RST \) are similar under \( PQR \leftrightarrow STR \), PQ = 12, PR = 15, \( \frac{QR}{TR} = \frac{3}{2} \). Find ST and SR. [2 marks]

13. In the map of a triangular field, sides are shown by 8 cm, 7 cm and 6 cm. If the largest side of the triangular field is 400 m, find the remaining sides of the field. [3 marks]

14. \( \triangle EFG \sim \triangle ARST \) and EF = 8, FG = 10, EG = 6, RS = 4. Find ST and RT. [2 marks]

15. In \( \square ABCD \), side BC \parallel \) side AD. Diagonals AC and BD intersect each other at P. If AP = \( \frac{1}{3} \) AC, then prove that DP = \( \frac{1}{2} \) BP. [Oct 09] [4 marks]

Based on Exercise 1.4

16. If \( \triangle PQR \sim \triangle PMN \) and 9A(\( \triangle PQR \)) = 6A(\( \triangle PMN \)), then find \( \frac{QR}{MN} \). [2 marks]

17. \( \triangle LMN \sim \triangle ARST \) and A(\( \triangle LMN \)) = 100 sq. cm, A(\( \triangle ARST \)) = 144 sq. cm, LM = 5 cm. Find RS. [2 marks]

18. \( \triangle ABC \) and \( \triangle DEF \) are equilateral triangles. A(\( \triangle ABC \)) : A(\( \triangle DEF \)) = 1 : 2 and AB = 4 cm. Find DE. [2 marks]

19. If the areas of two similar triangles are equal, then prove that they are congruent. [4 marks]

20. In the adjoining figure, seg DE \parallel \) side AB, DC = 2BD, A(\( \triangle CDE \)) = 20 cm\(^2\). Find A(\( \square ABDE \)). [4 marks]

Based on Exercise 1.5

21. In the adjoining figure, \( \angle PQR = 90^\circ \), seg QS \perp \) side PR. Find values of x, y and z. [3 marks]

22. In the adjoining figure, \( \angle PRQ = 90^\circ \), seg RS \perp \) seg PQ. Prove that: \( \frac{PR^2}{QR^2} = \frac{PS}{QS} \). [3 marks]
23. In the adjoining figure, 
\[ \angle PQR = 90^\circ, \]  
\[ \angle PSR = 90^\circ. \]  
Find: 
i. PR and ii. RS  [3 marks]

24. In the adjoining figure, 
\[ \square ABCD \text{ is a trapezium, } \text{seg AB} \parallel \text{seg DC}, \]  
\[ \text{seg DE} \perp \text{side AB}, \text{seg CF} \perp \text{side AB}. \]  
Find: i. DE and CF ii. BF iii. AB.  [5 marks]

25. Starting from Anil’s house, Peter first goes 50 m to south, then 75 m to west, then 62 m to North and finally 40 m to east and reaches Salim’s house. Then find the distance between Anil’s house and Salim’s house.  [5 marks]

26. In the adjoining figure, 
\[ \angle S = 90^\circ, \angle T = x^\circ, \]  
\[ \angle R = (x + 30)^\circ, \]  
RT = 16. 
Find: i. RS ii. ST  [3 marks]

27. \( \triangle DEF \) is an equilateral triangle. 
\[ \text{seg DP} \perp \text{side EF}, \]  
and \( E \equiv P \equiv F. \)  
Prove that : 
\[ \text{DP}^2 = 3 \text{EP}^2 \]  [Oct 08] [4 marks]

28. In the adjoining figure, 
\[ \square PQRV \text{ is a trapezium, } \text{seg PQ} \parallel \text{seg VR}. \]  
SR = 6, PQ= 9, Find VR.  [Mar 13] [3 marks]

29. In the adjoining figure, \( \triangle PQR \) is an equilateral triangle, 
\[ \text{seg PM} \perp \text{side QR}. \]  
Prove that: 
\[ \text{PQ}^2 = 4\text{QM}^2 \]  [3 marks]

Based on Exercise 1.6

26. In the adjoining figure, 
\[ \angle S = 90^\circ, \angle T = x^\circ, \]  
\[ \angle R = (x + 30)^\circ, \]  
RT = 16. 
Find: i. RS ii. ST  [3 marks]

27. \( \triangle DEF \) is an equilateral triangle. 
\[ \text{seg DP} \perp \text{side EF}, \]  
and \( E \equiv P \equiv F. \)  
Prove that : 
\[ \text{DP}^2 = 3 \text{EP}^2 \]  [Oct 08] [4 marks]

28. In the adjoining figure, 
\[ \square PQRV \text{ is a trapezium, } \text{seg PQ} \parallel \text{seg VR}. \]  
SR = 6, PQ= 9, Find VR.  [Mar 13] [3 marks]

Based on Exercise 1.7

30. In \( \triangle PQR \), seg PM is a median. PM = 10 and 
\[ \text{PQ}^2 + \text{PR}^2 = 362. \]  
Find QR.  [2 marks]

31. Adjacent sides of a parallelogram are 11 cm and 17 cm. Its one diagonal is 12 cm. Find its other diagonal.  [4 marks]

32. In \( \triangle ABC \), \( \angle ABC = 90^\circ \), AB = 12, BC = 16 and seg BP is a median. Find BP.  [3 marks]

Answers to additional problems for practice

1. i. \( \frac{1}{3} \) ii. \( \frac{2}{3} \) iii. \( \frac{2}{1} \)
2. 20 cm
3. i. \( \frac{AE}{DF} \) ii. \( \frac{BF}{FC} \) iii. \( \frac{EC \times AE}{BF \times DF} \)
4. 9 units
5. Yes, line MN \parallel \text{side PQ}
6. 20 units
7. seg MS \cong seg SN
8. 10 unit
9. PL = 28 units and NL = 30 units
10. ST = 8 units and SR = 10 units
11. Remaining sides of field are 350 m and 300 m.
12. ST = 5 units and RT = 3 units
13. 4 \( \frac{4}{3} \)
14. 6 cm
15. \( 4 \sqrt{2} \) cm
16. 25 cm\(^2\)
17. \( x = 4 \sqrt{5} \) units, \( y = 12 \) units and \( z = 6 \sqrt{5} \) units
18. \( 15 + 6 \sqrt{3} \) units
19. 37 m
20. 8 units ii. \( 8 \sqrt{3} \) units
21. 20 units
22. 18 units
23. 26 cm
24. 10 units