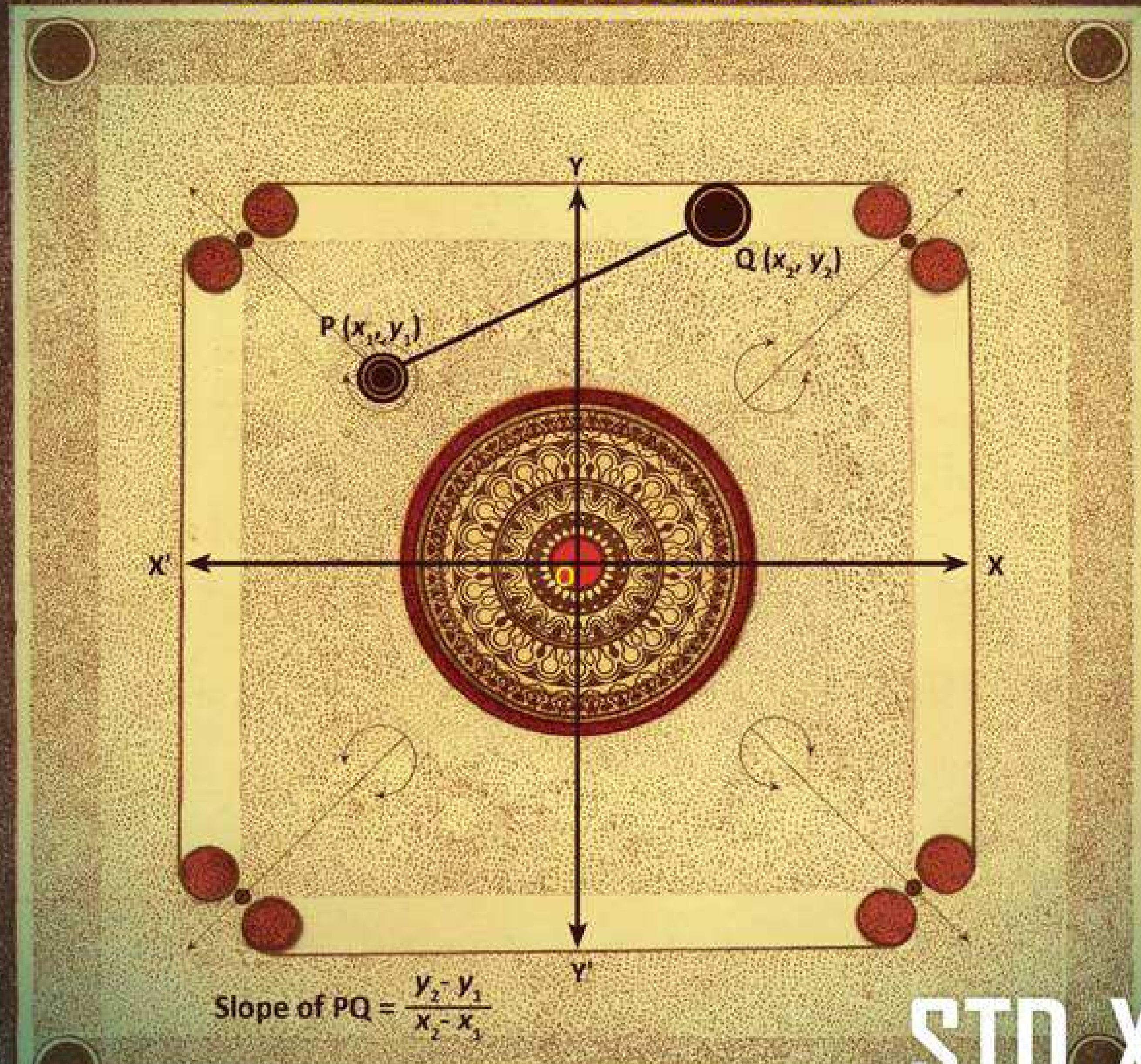
MATHEMATICS - II

GEOMETRY

BASED ON MAHARASHTRA STATE BOARD SYLLABUS



STD. X
(ENG. MED.)

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Target Publications Pvt. Ltd.

STD. X

Mathematics II

Geometry

Sixth Edition: March 2016

Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Topic–wise distribution of all textual questions and practice problems at the beginning of every chapter
- Covers solutions to all textual exercises and problem set.
- Includes additional problems for practice.
- Indicative marks for all problems.
- Comprehensive solution to Question Bank.
- Constructions drawn with accurate measurements.
- Includes Board Question Papers of 2014, 2015 and March 2016.

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Preface

Geometry is the mathematics of properties, measurement and relationships of points, lines, angles, surfaces and solids. It is widely used in the fields of science, engineering, computers, architecture etc. It is a vast subject dealing with the study of properties, definitions, theorems, areas, perimeter, angles, triangles, mensuration, co-ordinates, constructions etc.

The study of Geometry requires a deep and intrinsic understanding of concepts. Hence, to ease this task, we bring to you **"Std. X: Geometry",** a complete and thorough guide critically analysed and extensively drafted to boost the confidence of the students. The question answer format of this book helps the student to understand and grasp each and every concept thoroughly. The book is based on the new text book and covers the entire syllabus. At the beginning of every chapter, topic–wise distribution of all textual questions and practice problems has been provided for simpler understanding of different types of questions. The book contains answers to textual exercises, problems sets and Question bank. It also includes additional questions for practice. All the diagrams are neat and have proper labelling. The book has a unique feature that all the constructions are as per the scale. Another feature of the book is its layout which is attractive and inspires the student to read.

Marks are provided for each and every problem. However, marks mentioned are indicative and are subject to change as per Maharashtra State Board's discretion.

There is always room for improvement and hence we welcome all suggestions and regret any errors that may have occurred in the making of this book.

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

Yours' faithfully,

Publisher

MARKING SCHEME

Marking Scheme (for March 2014 exam and onwards)

Written Exam

Algebra	40 Marks	Time: 2 hrs.
Geometry	40 Marks	Time: 2 hrs.
* Internal Assessment	20 Marks	
Total	**100 Marks**	

* Internal Assessment		
Home Assignment:	10 Marks	5-5 Home assignment for Algebra and Geometry of 10 marks each would be given. Marks obtained out of 100 would be converted to marks out of 10.
Test of multiple choice question:	10 Marks	Depending upon the entire syllabus, internal test for Algebra and Geometry with 20 marks each would be taken at the end of second semester. Marks obtained out of 40 would be converted to marks out of 10.
Total	**20 marks**	

ALGEBRA AND GEOMETRY

Mark Wise Distribution of Questions

	Marks	Marks with Option
6 sub questions of 1 mark each: Attempt any 5	05	06
6 sub questions of 2 marks each: Attempt any 4	08	12
5 sub questions of 3 marks each: Attempt any 3	09	15
3 sub questions of 4 marks each: Attempt any 2	08	12
3 sub questions of 5 marks each: Attempt any 2	10	15
Total:	**40**	**60**

Weightage to Types of Questions

Sr. No.	Type of Questions	Marks	Percentage of Marks
1.	Very short answer	06	10
2.	Short answer	27	45
3 .	Long answer	27	45
	Total:	**60**	**100**

Weightage to Objectives

Sr. No	Objectives	Algebra Percentage marks	Geometry Percentage marks
1.	Knowledge	15	15
2.	Understanding	15	15
3.	Application	60	50
4.	Skill	10	20
	Total:	100	100

Unit wise Distribution: Algebra

Sr. No.	Unit	Marks with option
1.	Arithmetic Progression	12
2.	Quadratic equations	12
3.	Linear equation in two variables	12
4.	Probability	10
5.	Statistics – I	06
6.	Statistics – II	08
	Total:	**60**

Unit wise Distribution: Geometry

Sr. No.	Unit	Marks with option
1.	Similarity	12
2.	Circle	10
3.	Geometric Constructions	10
4.	Trigonometry	10
5.	Co-ordinate Geometry	08
6.	Mensuration	10
	Total:	**60**

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01 Similarity

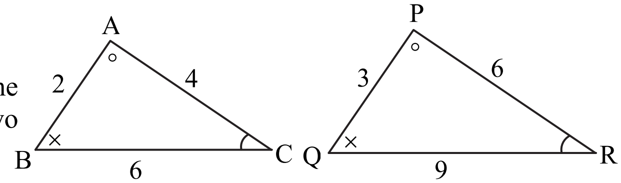
Type of Problems	Exercise	Q. Nos.
Properties of the Ratios of Areas of Two Triangles	1.1	Q.1, 2, 3, 4, 5, 6, 7
	Practice Problems (Based on Exercise 1.1)	Q.1, 2, 3
	Problem set-1	Q.7 (iii.), 20
Basic Proportionality Theorem (B.P.T.) and Converse of B.P.T.	1.2	Q.1, 2, 6, 10
	Practice Problems (Based on Exercise 1.2)	Q.4, 5, 6, 10
	Problem set-1	Q.6 (i.), 15, 18, 19, 21
Application of BPT (Property of Intercept made by Three Parallel lines on a Transversal and/or Property of an Angle Bisector of a Triangle)	1.2	Q.3, 4, 5, 7, 9
	Practice Problems (Based on Exercise 1.2)	Q.7, 8, 9
	Problem set-1	Q.16, 22
Similarity of Triangles	1.2	Q.8
	1.3	Q.1, 2, 3, 4, 5, 6
	Practice Problems (Based on Exercise 1.3)	Q.11, 12, 13, 14, 15
	Problem set-1	Q.1, 2, 4 (i., ii.), 7 (i., ii.), 8, 9, 10, 24, 25
Areas of Similar Triangles	1.4	Q.1, 2, 3, 4, 5, 6
	Practice Problems (Based on Exercise 1.4)	Q.16, 17, 18, 19, 20
	Problem set-1	Q.3, 4(iii.), 5, 6(ii., iii.), 17, 23
Similarity in Right Angled Triangles and Property of Geometric Mean	1.5	Q.2, 6 (i.)
	Practice Problems (Based on Exercise 1.5)	Q.22
	1.7	Q.4
Pythagoras Theorem and Converse of Pythagoras Theorem	1.5	Q.1, 3, 4, 5, 6(ii.), 7, 8
	Practice Problems (Based on Exercise 1.5)	Q.21, 23, 24, 25
	1.6	Q.2, 4
	Problem set-1	Q.11, 12
Theorem of 30°-60°-90° Triangle, Converse of 30°-60°-90° Triangle Theorem and Theorem of 45°-45°-90° Triangle	1.6	Q.1, 3, 5, 6, 7
	Practice Problems (Based on Exercise 1.6)	Q.26, 27, 28, 29
Applications of Pythagoras Theorem	1.7	Q.5
Apollonius Theorem	1.7	Q.1, 2, 3, 6
	Practice Problems (Based on Exercise 1.7)	Q.30, 31, 32
	Problem set-1	Q.13, 14

Concepts of Std. IX

Similarity of triangles

For a given one-to-one correspondence between the vertices of two triangles, if

i. their corresponding angles are congruent and
ii. their corresponding sides are in proportion then the correspondence is known as similarity and the two triangles are said to be similar.



In the figure, for correspondence ABC ↔ PQR,

i. $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

ii. $\frac{AB}{PQ} = \frac{2}{3}$, $\frac{BC}{QR} = \frac{6}{9} = \frac{2}{3}$, $\frac{AC}{PR} = \frac{4}{6} = \frac{2}{3}$

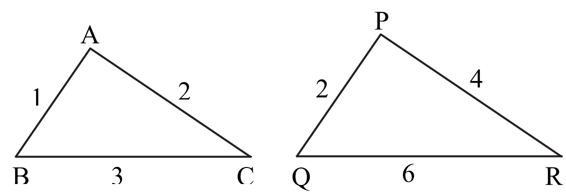
i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Hence, ΔABC and ΔPQR are similar triangles and are symbolically written as $\Delta ABC \sim \Delta PQR$.

Test of similarity of triangles

1. S–S–S test of similarity:

For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if the sides of one triangle are proportional to the corresponding sides of the other triangle.



In the figure,

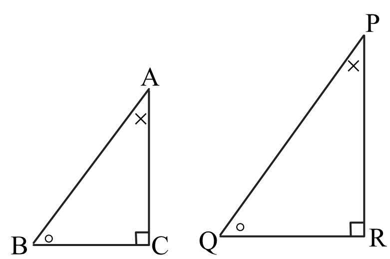
$\frac{AB}{PQ} = \frac{1}{2}$, $\frac{BC}{QR} = \frac{3}{6} = \frac{1}{2}$, $\frac{AC}{PR} = \frac{2}{4} = \frac{1}{2}$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\therefore \Delta ABC \sim \Delta PQR$ ---- [By S–S–S test of similarity]

2. A–A–A test of similarity [A–A test]:

For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if the angles of one triangle are congruent to the corresponding angles of the other triangle.



In the figure,

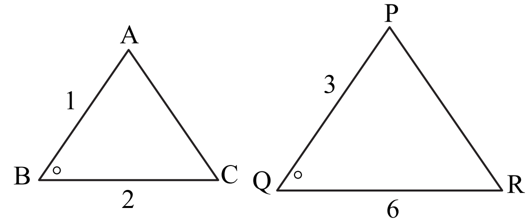
if $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

then $\Delta ABC \sim \Delta PQR$ ---- [By A–A–A test of similarity]

Note: A–A–A test is verified same as A–A test of similarity.

3. S–A–S test of similarity:

For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar if two sides of a triangle are proportional to the two corresponding sides of the other triangle and the corresponding included angles are also congruent.



In the figure,

$\frac{AB}{PQ} = \frac{1}{3}$, $\frac{BC}{QR} = \frac{2}{6} = \frac{1}{3}$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B \cong \angle Q$

$\therefore \Delta ABC \sim \Delta PQR$ ---- [By S–A–S test of similarity]

Converse of the test for similarity:

i. Converse of S–S–S test:

If two triangles are similar, then the corresponding sides are in proportion.

If $\Delta ABC \sim \Delta PQR$ then,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{---- [Corresponding sides of similar triangles]}$$

ii. Converse of A–A–A test:

If two triangles are similar, then the corresponding angles are congruent.

If $\Delta ABC \sim \Delta PQR$,

then $\angle A \cong \angle P$, $\angle B \cong \angle Q$ and $\angle C \cong \angle R$ ---- [Corresponding angles of similar triangles]

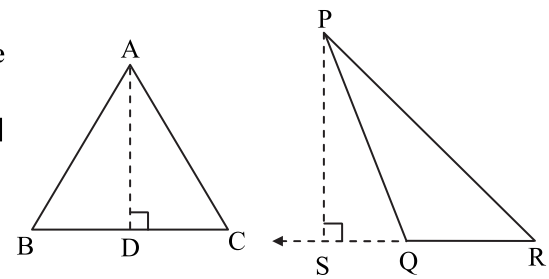
Note: 'Corresponding angles of similar triangles' can also be written as c.a.s.t.
'Corresponding sides of similar triangles' can also be written as c.s.s.t.

1.1 Properties of the ratios of areas of two triangles

Property – I

The ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights.

[2 marks]



Given: In ΔABC and ΔPQR, seg AD $\perp$ seg BC, B–D–C, seg PS $\perp$ ray RQ, S–Q–R

To prove that: $\dfrac{A(\Delta ABC)}{A(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS}$

Proof:

$A(\Delta ABC) = \dfrac{1}{2} \times BC \times AD$ ---- (i)

$A(\Delta PQR) = \dfrac{1}{2} \times QR \times PS$ ---- (ii)

[Area of a triangle = $\dfrac{1}{2}$ × base × height]

Dividing (i) by (ii), we get

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

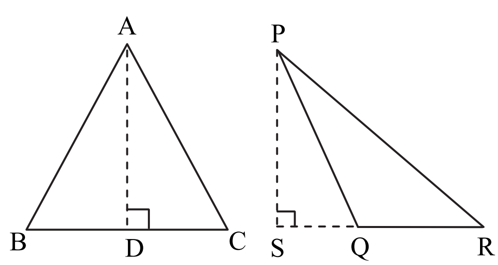
$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$

For Understanding

When do you say the triangles have equal heights?

We can discuss this in three cases.

Case – I

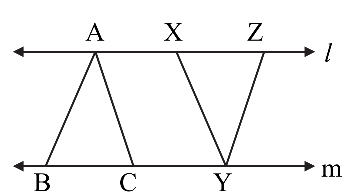


In the adjoining figure,

segments AD and PS are the corresponding heights of ΔABC and ΔPQR respectively.

If AD = PS, then ΔABC and ΔPQR are said to have equal heights.

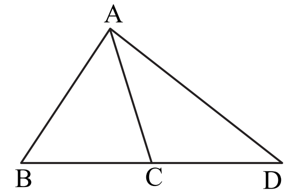
Case – II



In the adjoining figure, ΔABC and ΔXYZ have their one vertex on one of the parallel lines and the other two vertices lie on the other parallel line. Hence the two triangles are said to lie between the same parallel lines and are said to have equal heights.

Case – III

In the adjoining figure, ΔABC, ΔACD and ΔABD have a common vertex A and the sides opposite to vertex A namely, BC, CD and BD respectively of these triangles lie on the same line. Hence, ΔABC, ΔACD and ΔABD are said to have equal heights and BC, CD and BD are their respective bases.



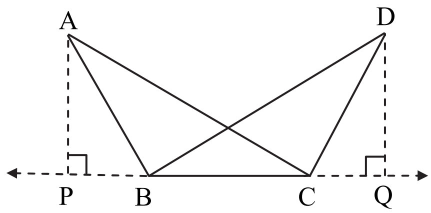
Property – II

The ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.

Example:

ΔABC and ΔDCB have a common base BC.

$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{AP}{DQ}$



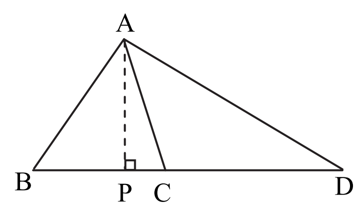
Property – III

The ratio of areas of two triangles having equal height is equal to the ratio of their corresponding bases.

Example:

ΔABC, ΔACD and ΔABD have a common vertex A and their sides opposite to vertex A namely, BC, CD, BD respectively lie on the same line. Hence they have equal heights. Here, AP is common height.

$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta ACD)} = \frac{BC}{CD}, \frac{A(\Delta ABC)}{A(\Delta ABD)} = \frac{BC}{BD}, \frac{A(\Delta ACD)}{A(\Delta ABD)} = \frac{CD}{BD}$



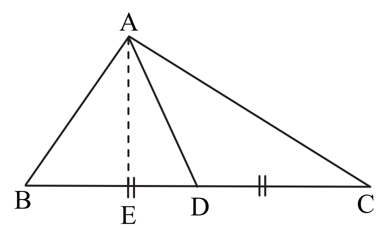
Property – IV

Areas of two triangles having equal bases and equal heights are equal.

Example:

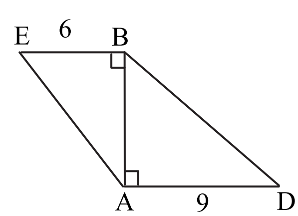
ΔABD and ΔACD have a common vertex A and their sides opposite to vertex A namely, BD and DC respectively lie on the same line. Hence the triangles have equal heights. Also their bases BD and DC are equal.

$\therefore \quad A(\Delta ABD) = A(\Delta ACD)$



Exercise 1.1

1. **In the adjoining figure, seg BE ⊥ seg AB and seg BA ⊥ seg AD. If BE = 6 and AD = 9, find $\frac{A(\Delta ABE)}{A(\Delta BAD)}$.** **[Oct 14, July 15] [1 mark]**



Solution:

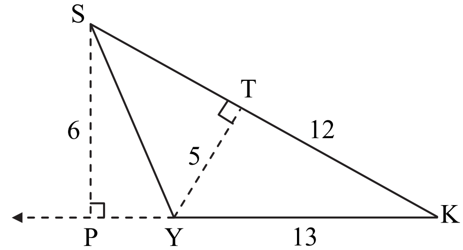
$\frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{BE}{AD}$ ---- [Ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.]

$\therefore \quad \frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{6}{9}$

$\therefore \quad \mathbf{\frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{2}{3}}$

2. In the adjoining figure, seg SP ⊥ side YK and seg YT ⊥ seg SK. If SP = 6, YK = 13, YT = 5 and TK = 12, then find A(ΔSYK) : A(ΔYTK).

[2 marks]



Solution:

$\frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{YK \times SP}{TK \times YT}$ ---- [Ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights.]

∴ $\frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{13 \times 6}{12 \times 5}$

∴ $\frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{13}{10}$

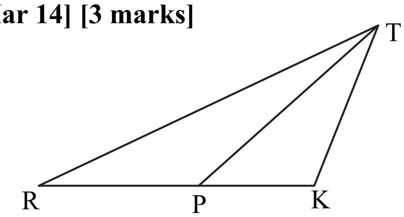
∴ **A(ΔSYK) : A(ΔYTK) = 13 : 10**

---

3. In the adjoining figure, RP : PK = 3 : 2, then find the values of the following ratios:

i. A(ΔTRP) : A(ΔTPK) **ii. A(ΔTRK) : A(ΔTPK)**

iii. A(ΔTRP) : A(ΔTRK) **[Mar 14] [3 marks]**



Solution:

RP : PK = 3 : 2 ---- [Given]

Let the common multiple be x.

∴ RP = $3x$, PK = $2x$ ---- (i)

RK = RP + PK ---- [R–P–K]

∴ RK = $3x + 2x$

∴ RK = $5x$ ---- (ii)

i. $\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{RP}{PK}$ ---- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

∴ $\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{3x}{2x}$ ---- [From (i)]

∴ $\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{3}{2}$

∴ **A(ΔTRP) : A(ΔTPK) = 3 : 2**

ii. $\frac{A(\Delta TRK)}{A(\Delta TPK)} = \frac{RK}{PK}$ ---- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

∴ $\frac{A(\Delta TRK)}{A(\Delta TPK)} = \frac{5x}{2x}$ ---- [From (i) and (ii)]

∴ $\frac{A(\Delta TRK)}{A(\Delta TPK)} = \frac{5}{2}$

∴ **A(ΔTRK) : A(ΔTPK) = 5 : 2**

iii. $\frac{A(\Delta TRP)}{A(\Delta TRK)} = \frac{RP}{RK}$ ---- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

∴ $\frac{A(\Delta TRP)}{A(\Delta TRK)} = \frac{3x}{5x}$ ---- [From (i) and (ii)]

∴ $\frac{A(\Delta TRP)}{A(\Delta TRK)} = \frac{3}{5}$

∴ **A(ΔTRP) : A(ΔTRK) = 3 : 5**

4. **The ratio of the areas of two triangles with the common base is 6 : 5. Height of the larger triangle is 9 cm. Then find the corresponding height of the smaller triangle. [Mar 15] [3 marks]**

Solution:

Let A_1 and A_2 be the areas of larger triangle and smaller triangle respectively and h_1 and h_2 be their corresponding heights.

$\frac{A_1}{A_2} = \frac{6}{5}$ ---- (i) [Given]

$h_1 = 9$ ---- (ii) [Given]

$\frac{A_1}{A_2} = \frac{h_1}{h_2}$ ---- [Ratio of the areas of two triangles having equal base is equal to the ratio of their corresponding heights.]

$\therefore \frac{6}{5} = \frac{9}{h_2}$ ---- [From (i) and (ii)]

$\therefore h_2 = \frac{5 \times 9}{6}$

$\therefore h_2 = \frac{15}{2}$

$\therefore h_2 = 7.5$ cm

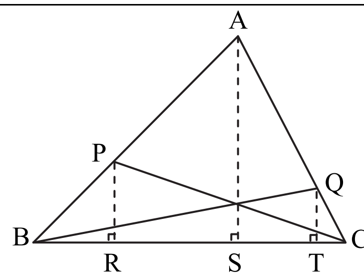
$\therefore$ **The corresponding height of the smaller triangle is 7.5 cm.**

---

5. **In the adjoining figure, seg PR ⊥ seg BC, seg AS ⊥ seg BC and seg QT ⊥ seg BC. Find the following ratios: [3 marks]**

i. $\frac{A(\Delta ABC)}{A(\Delta PBC)}$ **ii.** $\frac{A(\Delta ABS)}{A(\Delta ASC)}$

iii. $\frac{A(\Delta PRC)}{A(\Delta BQT)}$ **iv.** $\frac{A(\Delta BPR)}{A(\Delta CQT)}$



Solution:

i. $\frac{A(\Delta ABC)}{A(\Delta PBC)} = \frac{\mathbf{AS}}{\mathbf{PR}}$ ---- [Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.]

ii. $\frac{A(\Delta ABS)}{A(\Delta ASC)} = \frac{\mathbf{BS}}{\mathbf{SC}}$ ---- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

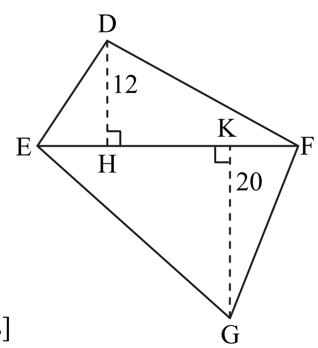
iii. $\frac{A(\Delta PRC)}{A(\Delta BQT)} = \frac{\mathbf{RC \times PR}}{\mathbf{BT \times QT}}$ ---- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]

iv. $\frac{A(\Delta BPR)}{A(\Delta CQT)} = \frac{\mathbf{BR \times PR}}{\mathbf{CT \times QT}}$ ---- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]

---

6. **In the adjoining figure, seg DH ⊥ seg EF and seg GK ⊥ seg EF. If DH = 12 cm, GK = 20 cm and A(ΔDEF) = 300 cm², then find**

i. EF ii. A(ΔGEF) iii. A(□DFGE) [3 marks]



Solution:

i. Area of triangle = $\frac{1}{2} \times$ base $\times$ height

$\therefore A(\Delta DEF) = \frac{1}{2} \times EF \times DH$

$\therefore 300 = \frac{1}{2} \times EF \times 12$ ---- [Substituting the given values]

$\therefore 300 = EF \times 6$

$\therefore EF = \frac{300}{6}$

$\therefore$ **EF = 50 cm**

ii. $\frac{A(\Delta DEF)}{A(\Delta GEF)} = \frac{DH}{GK}$ ---- [Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.]

$\therefore$ $\frac{300}{A(\Delta GEF)} = \frac{12}{20}$ ---- [Substituting the given values]

$\therefore$ $300 \times 20 = 12 \times A(\Delta GEF)$

$\therefore$ $\frac{300 \times 20}{12} = A(\Delta GEF)$

$\therefore$ $A(\Delta GEF) = \frac{300 \times 20}{12}$

$\therefore$ **$A(\Delta GEF) = 500\ cm^2$** ---- (i)

---

iii. $A(\square DFGE) = A(\Delta DEF) + A(\Delta GEF)$ ---- [Area addition property]

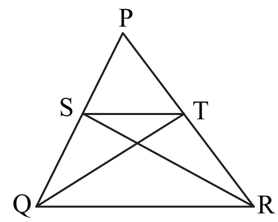
$\therefore$ $A(\square DFGE) = 300 + 500$ ---- [From (i) and given]

$\therefore$ **$A(\square DFGE) = 800\ cm^2$**

---

7. In the adjoining figure, seg ST || side QR. Find the following ratios. [3 marks]

i. $\frac{A(\Delta PST)}{A(\Delta QST)}$ **ii.** $\frac{A(\Delta PST)}{A(\Delta RST)}$ **iii.** $\frac{A(\Delta QST)}{A(\Delta RST)}$



Solution:

i. $\frac{A(\Delta PST)}{A(\Delta QST)} = \frac{\mathbf{PS}}{\mathbf{QS}}$

ii. $\frac{A(\Delta PST)}{A(\Delta RST)} = \frac{\mathbf{PT}}{\mathbf{TR}}$

[Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

iii. ΔQST and ΔRST lie between the same parallel lines ST and QR

$\therefore$ Their heights are equal.

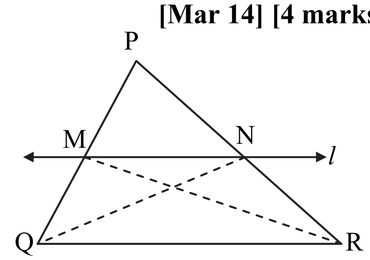
Also ST is the common base.

$\therefore$ $A(\Delta QST) = A(\Delta RST)$ ---- [Areas of two triangles having common base and equal heights are equal.]

$\therefore$ $\frac{A(\Delta QST)}{A(\Delta RST)} = \mathbf{1}$

1.2 Basic Proportionality Theorem (B.P.T)

If a line parallel to a side of a triangle intersects the other sides in two distinct points, then the line divides these sides in proportion. **[Mar 14] [4 marks]**



Given: In ΔPQR, line l || side QR.
Line l intersects side PQ and side PR in points M and N respectively, such that P–M–Q and P–N–R.

To Prove that: $\frac{PM}{MQ} = \frac{PN}{NR}$

Construction: Draw seg QN and seg RM.

Proof:

In ΔPMN and ΔQMN, where P–M–Q,

$\frac{A(\Delta PMN)}{A(\Delta QMN)} = \frac{PM}{MQ}$ ---- (i) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

In ΔPMN and ΔRMN, where P–N–R,

$\frac{A(\Delta PMN)}{A(\Delta RMN)} = \frac{PN}{NR}$ ---- (ii) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

$A(\Delta QMN) = A(\Delta RMN)$ ---- (iii) [Areas of two triangles having equal bases and equal heights are equal.]

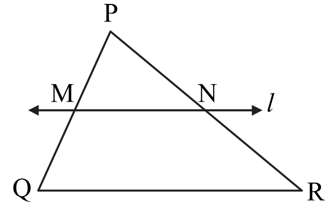
$\therefore$ $\frac{A(\Delta PMN)}{A(\Delta QMN)} = \frac{A(\Delta PMN)}{A(\Delta RMN)}$ ---- (iv) [From (i), (ii) and (iii)]

$\therefore$ $\frac{PM}{MQ} = \frac{PN}{NR}$ ---- [From (i), (ii) and (iv)]

Converse of Basic Proportionality Theorem:

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If line l intersects the side PQ and side PR of ΔPQR in the points M and N respectively such that $\frac{PM}{MQ} = \frac{PN}{NR}$, then line l || side QR.



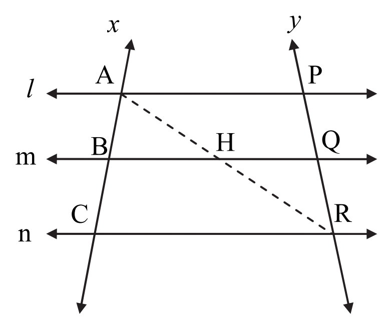
Applications of Basic Proportionality Theorem:

i. Property of intercepts made by three parallel lines on a transversal:
The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same three parallel lines. [3 marks]

Given: line l || line m || line n
The transversals x and y intersect these parallel lines at points A, B, C and P, Q, R respectively.

To Prove that: $\frac{AB}{BC} = \frac{PQ}{QR}$

Construction: Draw seg AR to intersect line m at point H.



Proof:

In ΔACR,
seg BH || side CR ---- [Given]
∴ $\frac{AB}{BC} = \frac{AH}{HR}$ ---- (i) [By B.P.T.]
In ΔARP,
seg HQ || side AP ---- [Given]
$\frac{QR}{PQ} = \frac{RH}{HA}$ ---- [By B.P.T.]
∴ $\frac{PQ}{QR} = \frac{AH}{HR}$ ---- (ii) [By invertendo]
∴ $\frac{AB}{BC} = \frac{PQ}{QR}$ ---- [From (i) and (ii)]

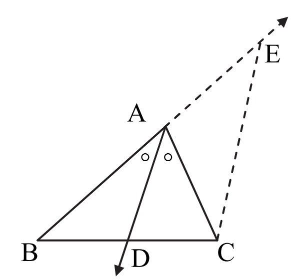
---

ii. Property of an angle bisector of a triangle:
In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides. [Mar 15] [5 marks]

Given: In ΔABC, ray AD bisects ∠BAC

To Prove that: $\frac{BD}{DC} = \frac{AB}{AC}$

Construction: Draw a line parallel to ray AD, passing through point C.
Extend BA to intersect the line at E.



Proof:

In ΔBEC,
seg AD || side EC ---- [By construction]
∴ $\frac{BD}{DC} = \frac{AB}{AE}$ ---- (i) [By B.P.T.]
line AD || line EC on transversal BE
∴ ∠BAD ≅ ∠AEC ---- (ii) [Corresponding angles]
line AD || line EC on transversal AC.
∴ ∠CAD ≅ ∠ACE ---- (iii) [Alternate angles]
Also, ∠BAD ≅ ∠CAD ---- (iv) [∵ Ray AD bisects ∠BAC]
∴ ∠AEC ≅ ∠ACE ---- (v) [From (ii), (iii) and (iv)]

In ΔAEC,

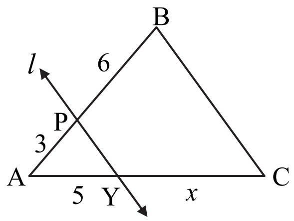
∠AEC ≅ ∠ACE ---- [From (v)]

∴ AE = AC ---- (vi) [Sides opposite to congruent angles]

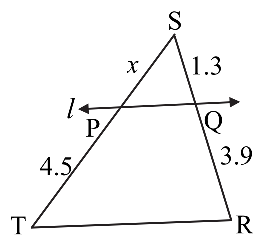
∴ $\frac{BD}{DC} = \frac{AB}{AC}$ ---- [From (i) and (vi)]

Exercise 1.2

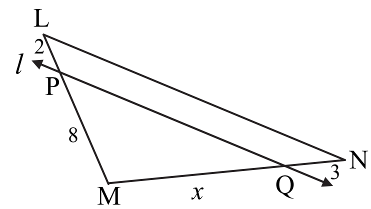
1. Find the values of x in the following figures, if line l is parallel to one of the sides of the given triangles.
[Oct 12, Mar 13] [1 mark each]



(i)



(ii)



(iii)

Solution:

i. In ΔABC,

line l || side BC ---- [Given]

∴ $\frac{AP}{PB} = \frac{AY}{YC}$ ---- [By B.P.T.]

∴ $\frac{3}{6} = \frac{5}{x}$

∴ $x = \frac{6 \times 5}{3}$

∴ **x = 10 units**

---

ii. In ΔRST,

line l || side TR ---- [Given]

$\frac{SP}{PT} = \frac{SQ}{QR}$ ---- [By B.P.T.]

∴ $\frac{x}{4.5} = \frac{1.3}{3.9}$

∴ $x = \frac{1.3 \times 4.5}{3.9}$

∴ $x = \frac{13 \times 45}{39 \times 10}$

∴ **x = 1.5 units**

---

iii. In ΔLMN,

line l || side LN ---- [Given]

∴ $\frac{MP}{PL} = \frac{MQ}{QN}$ ---- [By B.P.T.]

∴ $\frac{8}{2} = \frac{x}{3}$

∴ $\frac{3 \times 8}{2} = x$

∴ $x = 3 \times 4$

∴ **x = 12 units**

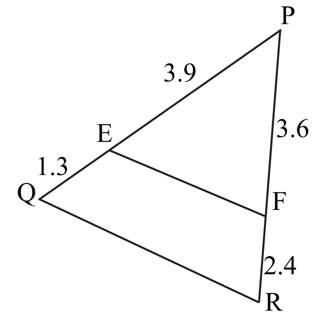
2. E and F are the points on the side PQ and PR respectively of ΔPQR. For each of the following cases, state whether EF || QR. [2 marks each]

i. PE = 3.9 cm, EQ = 1.3 cm, PF = 3.6 cm and FR = 2.4 cm.

ii. PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm.

iii. PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

Solution:



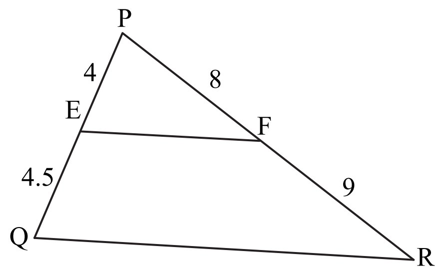
i. $\frac{PE}{EQ} = \frac{3.9}{1.3} = \frac{3}{1}$ ---- (i)

$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2}$ ---- (ii)

∴ In ΔPQR,

$\frac{PE}{EQ} \neq \frac{PF}{FR}$ ---- [From (i) and (ii)]

∴ **seg EF is not parallel to seg QR.**



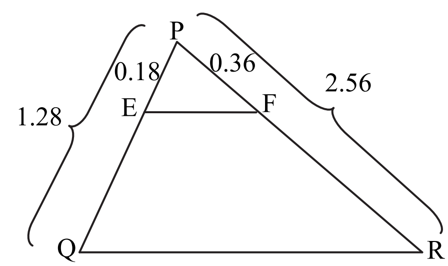
ii. $\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9}$ ---- (i)

$\frac{PF}{FR} = \frac{8}{9}$ ---- (ii)

In ΔPQR,

$\frac{PE}{QE} = \frac{PF}{FR}$ ---- [From (i) and (ii)]

∴ **seg EF || seg QR** ---- [By converse of B.P.T.]



iii. EQ + PE = PQ ---- [P–E–Q]

∴ EQ = PQ – PE

= 1.28 – 0.18 = 1.10

FR + PF = PR ---- [P–F–R]

∴ FR = PR – PF

= 2.56 – 0.36 = 2.20

∴ $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$ ---- (i)

$\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ ---- (ii)

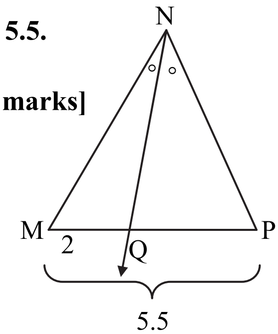
In ΔPQR,

$\frac{PE}{EQ} = \frac{PF}{FR}$ ---- [From (i) and (ii)]

∴ **seg EF || side QR** ---- [By converse of B.P.T.]

---

3. In the adjoining figure, point Q is on the side MP such that MQ = 2 and MP = 5.5. Ray NQ is the bisector of ∠MNP of ΔMNP. Find MN : NP. [2 marks]



Solution:

QP + MQ = MP ---- [M–Q–P]

∴ QP + 2 = 5.5

∴ QP = 5.5 – 2

∴ QP = 3.5

In ΔMNP,

ray NQ is the angle bisector of ∠MNP ---- [Given]

∴ $\frac{MN}{NP} = \frac{MQ}{QP}$ ---- [By property of angle bisector of a triangle]

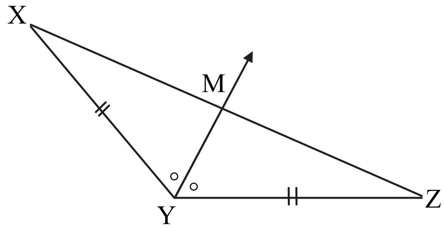
$\therefore \quad \frac{MN}{NP} = \frac{2}{3.5} = \frac{20}{35} = \frac{4}{7}$

$\therefore \quad \frac{MN}{NP} = \frac{4}{7}$

$\therefore$ **MN : NP = 4 : 7**

---

4. In the adjoining figure, ray YM is the bisector of ∠XYZ, where XY ≅ YZ. Find the relation between XM and MZ. [2 marks]



Solution:

In ΔXYZ,

Ray YM is the angle bisector of ∠XYZ ---- [Given]

$\therefore \quad \frac{XM}{MZ} = \frac{XY}{YZ}$ ---- (i) [By property of angle bisector of a triangle]

seg XY ≅ seg YZ ---- [Given]

$\therefore$ XY = YZ

$\therefore \quad \frac{XY}{YZ} = 1$ ---- (ii)

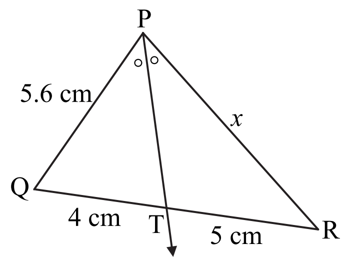
$\therefore \quad \frac{XM}{MZ} = 1$ ---- [From (i) and (ii)]

$\therefore$ XM = MZ

$\therefore$ **seg XM ≅ seg MZ**

---

5. In the adjoining figure, ray PT is the bisector of ∠QPR. Find the value of *x* and the perimeter of ΔPQR. [Mar 14] [3 marks]



Solution:

In ΔPQR,

Ray PT is the angle bisector of ∠QPR.

$\therefore \quad \frac{PQ}{PR} = \frac{QT}{TR}$ ---- [By property of angle bisector of a triangle]

$\therefore \quad \frac{5.6}{x} = \frac{4}{5}$

$\therefore \quad 5.6 \times 5 = 4 \times x$

$\therefore \quad \frac{5.6 \times 5}{4} = x$

$\therefore \quad x = 7$ cm

$\therefore$ PR = 7 cm ---- [$\because$ PR = x]

Now, QR = QT + TR ---- [Q–T–R]

$\therefore$ QR = 4 + 5

$\therefore$ QR = 9 cm

Perimeter of ΔPQR = PQ + QR + PR

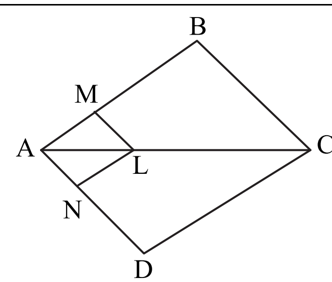
= 5.6 + 9 + 7 = 21.6 cm

$\therefore$ **The value of *x* is 7 cm and the perimeter of ΔPQR is 21.6 cm.**

---

6. In the adjoining figure, if ML || BC and NL || DC.

Then prove that $\frac{AM}{AB} = \frac{AN}{AD}$. [3 marks]



Proof:

In ΔABC,

seg ML || side BC ---- [Given]

$\therefore \quad \frac{AM}{MB} = \frac{AL}{LC}$ ---- (i) [By B.P.T.]

In ΔADC,

seg NL || side DC ---- [Given]

$\therefore \quad \frac{AN}{ND} = \frac{AL}{LC}$ ---- (ii) [By B.P.T.]

$\therefore \quad \frac{AM}{MB} = \frac{AN}{ND}$ ---- [From (i) and (ii)]

$\therefore \quad \frac{MB}{AM} = \frac{ND}{AN}$ ---- [By invertendo]

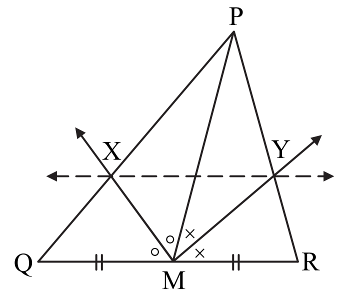
$\therefore \quad \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$ ---- [By componendo]

$\therefore \quad \frac{AB}{AM} = \frac{AD}{AN}$ ---- [A–M–B, A–N–D]

$\therefore \quad \mathbf{\frac{AM}{AB} = \frac{AN}{AD}}$ ---- [By invertendo]

---

7. As shown in the adjoining figure, in ΔPQR, seg PM is the median. Bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively, then prove that XY || QR. [3 marks]



Proof:

Draw line XY.

In ΔPMQ,

ray MX is the angle bisector of ∠PMQ. ---- [Given]

$\therefore \quad \frac{MP}{MQ} = \frac{PX}{QX}$ ---- (i) [By property of angle bisector of a triangle]

In ΔPMR,

ray MY is the angle bisector of ∠PMR. ---- [Given]

$\therefore \quad \frac{MP}{MR} = \frac{PY}{RY}$ ---- (ii) [By property of angle bisector of a triangle]

But, seg PM is the median ---- [Given]

∴ M is midpoint of seg QR.

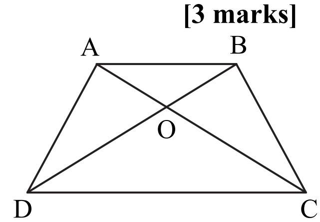
∴ MQ = MR ---- (iii)

$\therefore \quad \frac{PX}{QX} = \frac{PY}{RY}$ ---- [From (i), (ii) and (iii)]

In ΔPQR, seg XY || seg QR ---- [By converse of B.P.T.]

---

8. □ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\mathbf{\frac{AO}{BO} = \frac{CO}{DO}}$. [3 marks]



Proof:

□ABCD is a trapezium.

side AB || side DC and seg AC is a transversal.

∠BAC ≅ ∠DCA ---- (i) [Alternate angles]

In ΔAOB and ΔCOD,

∠BAO ≅ ∠DCO ---- [From (i) and A–O–C]

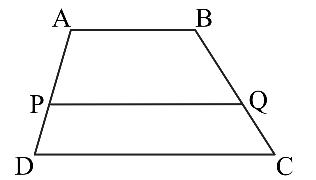
∠AOB ≅ ∠COD ---- [Vertically opposite angles]

∴ ΔAOB ~ ΔCOD ---- [By A–A test of similarity]

$\therefore \quad \frac{AO}{CO} = \frac{BO}{DO}$ ---- [c.s.s.t.]

$\therefore \quad \mathbf{\frac{AO}{BO} = \frac{CO}{DO}}$ ---- [By alternendo]

9. In the adjoining figure, □ABCD is a trapezium.
Side AB || seg PQ || side DC and AP = 15, PD = 12, QC = 14, then find BQ. [2 marks]



Solution:

Side AB || seg PQ || side DC ---- [Given]

$\therefore \frac{AP}{PD} = \frac{BQ}{QC}$ ---- [By property of intercepts made by three parallel lines on a transversal]

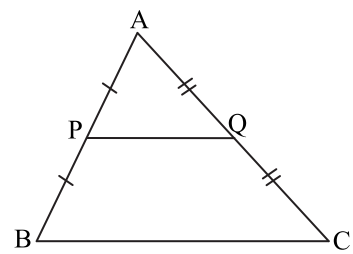
$\therefore \frac{15}{12} = \frac{BQ}{14}$ ---- [$\because$ AP = 15, PD = 12 and QC = 14]

$\therefore BQ = \frac{15 \times 14}{12}$

$\therefore$ **BQ = 17.5**

---

10. Using the converse of Basic Proportionality Theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side and is half of it. [4 marks]



Given: In ΔABC, P and Q are midpoints of sides AB and AC respectively.

To Prove: seg PQ || side BC

$PQ = \frac{1}{2}BC$

Proof:

AP = PB ---- [P is the midpoint of side AB.]

$\therefore \frac{AP}{PB} = 1$ ---- (i)

AQ = QC ---- [Q is the midpoint of side AC.]

$\therefore \frac{AQ}{QC} = 1$ ---- (ii)

In ΔABC,

$\frac{AP}{PB} = \frac{AQ}{QC}$ ---- [From (i) and (ii)]

$\therefore$ **seg PQ || side BC** ---- (iii) [By converse of B.P.T.]

In ΔABC and ΔAPQ,

$\angle ABC \cong \angle APQ$ ---- [From (iii), corresponding angles]

$\angle BAC \cong \angle PAQ$ ---- [Common angle]

$\therefore \Delta ABC \sim \Delta APQ$ ---- [By A–A test of similarity]

$\therefore \frac{AB}{AP} = \frac{BC}{PQ}$ ---- [c.s.s.t.]

$\therefore \frac{AP + PB}{AP} = \frac{BC}{PQ}$ ---- [A–P–B]

$\therefore \frac{AP + AP}{AP} = \frac{BC}{PQ}$ ---- [$\because$ AP = PB]

$\therefore \frac{2AP}{AP} = \frac{BC}{PQ}$

$\therefore \frac{2}{1} = \frac{BC}{PQ}$

$\therefore$ $\mathbf{PQ = \frac{1}{2}BC}$

1.3 Similarity

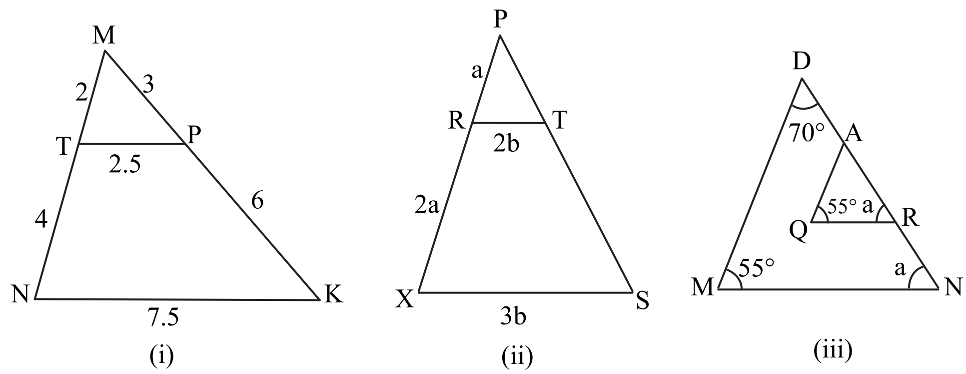
Two figures are called similar if they have same shapes not necessarily the same size.

Properties of Similar Triangles:

1. **Reflexivity:** ΔABC ~ ΔABC. It means a triangle is similar to itself.
2. **Symmetry:** If ΔABC ~ ΔDEF, then ΔDEF ~ ΔABC.
3. **Transitivity:** If ΔABC ~ ΔDEF and ΔDEF ~ ΔPQR, then ΔPQR ~ ΔABC.

Exercise 1.3

1. **Study the following figures and find out in each case whether the triangles are similar. Give reason.** **[2 marks each]**



Solution:

i. ΔMTP and ΔMNK are similar.

Reason:

MN = MT + TN ---- [M–T–N]

∴ MN = 2 + 4 = 6 units

∴ $\frac{MT}{MN} = \frac{2}{6} = \frac{1}{3}$ ---- (i)

MK = MP + PK ---- [M–P–K]

∴ MK = 3 + 6 = 9 units

∴ $\frac{MP}{MK} = \frac{3}{9} = \frac{1}{3}$ ---- (ii)

In ΔMTP and ΔMNK,

$\frac{MT}{MN} = \frac{MP}{MK}$ ---- [From (i) and (ii)]

∠TMP ≅ ∠NMK ---- [Common angle]

∴ **ΔMTP ~ ΔMNK** ---- [By S–A–S test of similarity]

---

ii. ΔPRT and ΔPXS are not similar.

Reason:

PX = PR + RX ---- [P–R–X]

∴ PX = a + 2a = 3a

∴ $\frac{PR}{PX} = \frac{a}{3a} = \frac{1}{3}$ ---- (i)

$\frac{RT}{XS} = \frac{2b}{3b} = \frac{2}{3}$ ---- (ii)

∴ $\frac{PR}{PX} \neq \frac{RT}{XS}$ ---- [From (i) and (ii)]

∴ The corresponding sides of the two triangles are not in proportion.

∴ **ΔPRT and ΔPXS are not similar.**

iii. **ΔDMN and ΔAQR are similar.**

Reason:

In ΔDMN and ΔAQR,

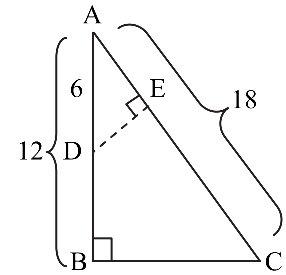
∠DMN ≅ ∠AQR ---- [Each is 55°]

∠DNM ≅ ∠ARQ ---- [Each is of same measure]

∴ **ΔDMN ~ ΔAQR** ---- [By A–A test of similarity]

---

2. **In the adjoining figure, ΔABC is right angled at B. D is any point on AB. seg DE ⊥ seg AC. If AD = 6 cm, AB = 12 cm, AC = 18 cm. Find AE.** **[2 marks]**



Solution:

In ΔAED and ΔABC,

∠AED ≅ ∠ABC ---- [Each is 90°]

∠DAE ≅ ∠BAC ---- [Common angle]

∴ ΔAED ~ ΔABC ---- [By A–A test of similarity]

∴ $\frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$ ---- [c.s.s.t.]

∴ $\frac{AE}{AB} = \frac{AD}{AC}$

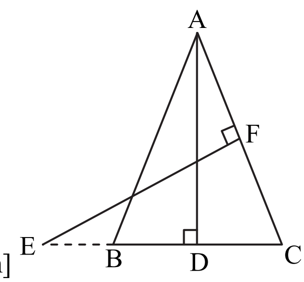
∴ $\frac{AE}{12} = \frac{6}{18}$

∴ $AE = \frac{6 \times 12}{18}$

∴ **AE = 4 cm**

---

3. **In the adjoining figure, E is a point on side CB produced of an isosceles ΔABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.** **[3 marks]**



Proof:

In ΔABC,

seg AB ≅ seg AC ---- [Given]

∠B ≅ ∠C ---- (i) [By isosceles triangle theorem]

In ΔABD and ΔECF,

∠ABD ≅ ∠ECF ---- [From (i)]

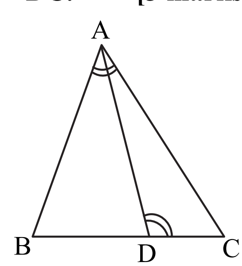
∠ADB ≅ ∠EFC ---- [Each is 90°]

∴ **ΔABD ~ ΔECF** ---- [By A–A test of similarity]

---

4. **D is a point on side BC of ΔABC such that ∠ADC = ∠BAC. Show that AC^2 = BC × DC.** **[3 marks]**

Proof:



In ΔACB and ΔDCA,

∠BAC ≅ ∠ADC ---- [Given]

∠ACB ≅ ∠DCA ---- [Common angle]

∴ ΔACB ~ ΔDCA ---- [By A–A test of similarity]

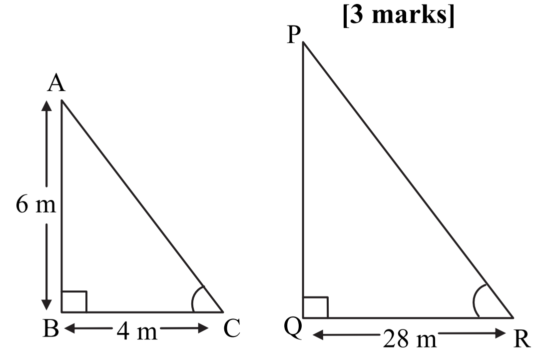
∴ $\frac{AC}{DC} = \frac{BC}{AC} = \frac{AB}{DA}$ ---- [c.s.s.t.]

∴ $\frac{AC}{DC} = \frac{BC}{AC}$

∴ **AC^2 = BC × DC**

5. A vertical pole of length 6 m casts a shadow of 4 m long on the ground. At the same time, a tower casts a shadow 28 m long. Find the height of the tower. **[3 marks]**

Solution:



AB represents the length of the pole.

$\therefore$ AB = 6 m

BC represents the shadow of the pole.

$\therefore$ BC = 4 m

PQ represents the height of the tower.

QR represents the shadow of the tower.

$\therefore$ QR = 28 m

$\Delta ABC \sim \Delta PQR$

---- [$\because$ vertical pole and tower are similar figures]

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ---- [c.s.s.t.]

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore$ $\frac{6}{PQ} = \frac{4}{28}$

$\therefore$ $\frac{6}{PQ} = \frac{1}{7}$

$\therefore$ $6 \times 7 = PQ$

$\therefore$ PQ = 42 m

$\therefore$ **Height of the tower is 42 m.**

---

6. Triangle ABC has sides of length 5, 6 and 7 units while ΔPQR has perimeter of 360 units. If ΔABC is similar to ΔPQR, then find the sides of ΔPQR. **[3 marks]**

Solution:

Since, $\Delta ABC \sim \Delta PQR$

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ---- [c.s.s.t.]

$\therefore$ $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR}$

By theorem on equal ratios,

each ratio = $\frac{5+6+7}{PQ+QR+PR}$

$= \frac{18}{360}$ ---- [$\because$ Perimeter of ΔPQR = PQ + QR + PR= 360]

$= \frac{1}{20}$

$\therefore$ $\frac{5}{PQ} = \frac{6}{QR} = \frac{7}{PR} = \frac{1}{20}$ ---- (i)

$\frac{5}{PQ} = \frac{1}{20}$ ---- [From (i)]

$\therefore$ PQ $= 20 \times 5$

$\therefore$ PQ = 100 units

$\frac{6}{QR} = \frac{1}{20}$ ---- [From (i)]

$\therefore$ QR $= 6 \times 20$

$\therefore$ QR = 120 units

$\frac{7}{PR} = \frac{1}{20}$ ---- [From (i)]

$\therefore$ PR $= 7 \times 20$

$\therefore$ PR = 140 units

$\therefore$ **ΔPQR has sides PQ, QR and PR of length 100 units, 120 units and 140 units respectively.**

iii. $\frac{A(\Delta PBC)}{A(\Delta PQA)} = \frac{25}{1}$ ---- [By invertendo]

∴ $\frac{A(\Delta PBC) - A(\Delta PQA)}{A(\Delta PQA)} = \frac{25-1}{1}$ ---- [By dividendo]

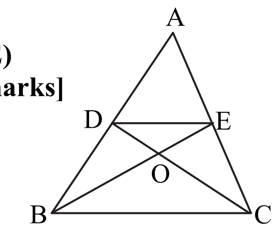
∴ $\frac{A(\square QBCA)}{A(\Delta PQA)} = \frac{24}{1}$

∴ $\frac{A(\Delta PQA)}{A(\square QBCA)} = \frac{1}{24}$ ---- [By invertendo]

∴ **A(ΔPQA) : A(□QBCA) = 1 : 24**

---

7. In the adjoining figure, DE || BC and AD : DB = 5 : 4.
Find: i. DE : BC ii. DO : DC iii. A(ΔDOE) : A(ΔDCE)
[5 marks]



Solution:

i. DE || BC ---- [Given]
AB is a transversal
∴ ∠ADE ≅ ∠ABC ---- (i) [Corresponding angles]
In ΔADE and ΔABC,
∠ADE ≅ ∠ABC ---- [From (i)]
∠DAE ≅ ∠BAC ---- [Common angle]
∴ ΔADE ~ ΔABC ---- [By A–A test of similarity]

∴ $\frac{AD}{AB} = \frac{DE}{BC}$ ---- (ii) [c.s.s.t.]

$\frac{AD}{DB} = \frac{5}{4}$ ---- [Substituting the given values]

∴ $\frac{DB}{AD} = \frac{4}{5}$ ---- [By invertendo]

∴ $\frac{DB + AD}{AD} = \frac{4+5}{5}$ ---- [By componendo]

∴ $\frac{AB}{AD} = \frac{9}{5}$ ---- [A–D–B]

∴ $\frac{AD}{AB} = \frac{5}{9}$ ---- (iii) [By invertendo]

∴ $\frac{DE}{BC} = \frac{5}{9}$ ---- (iv) [From (ii) and (iii)]

∴ **DE : BC = 5 : 9**

---

ii. In ΔDOE and ΔCOB,
∠EDO ≅ ∠BCO ---- [Alternate angles on parallel lines DE and BC]
∠DOE ≅ ∠COB ---- [Vertically opposite angles]
∴ ΔDOE ~ ΔCOB ---- [By A–A test of similarity]

∴ $\frac{DO}{OC} = \frac{DE}{BC}$ ---- [c.s.s.t.]

∴ $\frac{DO}{OC} = \frac{5}{9}$ ---- [From (iv)]

∴ $\frac{OC}{DO} = \frac{9}{5}$ ---- [By invertendo]

∴ $\frac{OC + DO}{DO} = \frac{9+5}{5}$ ---- [By componendo]

∴ $\frac{DC}{DO} = \frac{14}{5}$ ---- [D–O–C]

∴ $\frac{DO}{DC} = \frac{5}{14}$ ---- (v) [By invertendo]

∴ **DO : DC = 5 : 14**

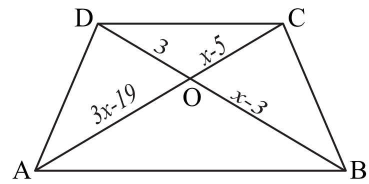
iii. $\frac{A(\Delta DOE)}{A(\Delta DCE)} = \frac{DO}{DC}$ ---- [Ratio of areas of two triangles having equal heights is equal to the ratio of the corresponding bases]

$\therefore$ $\frac{A(\Delta DOE)}{A(\Delta DCE)} = \frac{5}{14}$ ---- [From (v)]

$\therefore$ **A(ΔDOE) : A(ΔDCE) = 5 : 14**

---

8. In the adjoining figure, seg AB || seg DC.
Using the information given, find the value of *x*. **[3 marks]**



Solution:

Side DC || Side AB on transversal DB.

$\therefore$ $\angle ABD \cong \angle CDB$ ---- (i) [Alternate angles]

In ΔAOB and ΔCOD,

$\angle ABO \cong \angle CDO$ ---- [From (i), D – O – B]

$\angle AOB \cong \angle COD$ ---- [Vertically opposite angles]

$\therefore$ $\Delta AOB \sim \Delta COD$ ---- [By A–A test of similarity]

$\therefore$ $\frac{OA}{OC} = \frac{OB}{OD}$ ---- [c.s.s.t]

$\therefore$ $\frac{3x - 19}{x - 5} = \frac{x - 3}{3}$ ---- [Substituting the given values]

$\therefore$ $3(3x - 19) = (x - 3)(x - 5)$

$\therefore$ $9x - 57 = x^2 - 8x + 15$

$\therefore$ $x^2 - 8x - 9x + 15 + 57 = 0$

$\therefore$ $x^2 - 17x + 72 = 0$

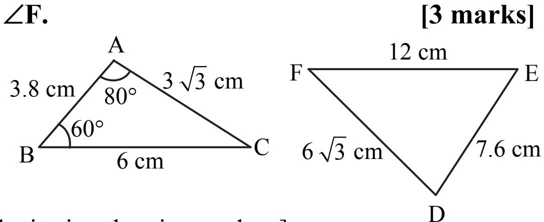
$\therefore$ $(x - 9)(x - 8) = 0$

$\therefore$ $x - 9 = 0$ or $x - 8 = 0$

$\therefore$ $\boldsymbol{x = 9}$ **or** $\boldsymbol{x = 8}$

---

9. Using the information given in the adjoining figure, find ∠F. **[3 marks]**



Solution:

$\frac{AB}{DE} = \frac{3.8}{7.6} = \frac{1}{2}$ ---- (i)

$\frac{BC}{EF} = \frac{6}{12} = \frac{1}{2}$ ---- (ii) [Substituting the given values]

$\frac{CA}{FD} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$ ---- (iii)

In ΔABC and ΔDEF,

$\therefore$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ---- [From (i), (ii) and (iii)]

$\therefore$ $\Delta ABC \sim \Delta DEF$ ---- [By S–S–S test of similarity]

$\angle C \cong \angle F$ ---- (iv) [c.a.s.t]

In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$ ---- [Sum of the measures of all angles of a triangle is 180°.]

$\therefore$ $80^\circ + 60^\circ + \angle C = 180^\circ$ ---- [Substituting the given values]

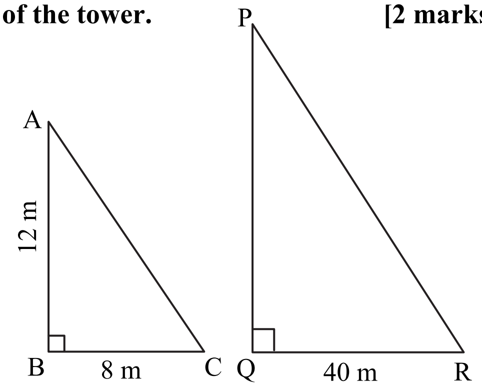
$\therefore$ $\angle C = 180^\circ - 140^\circ$

$\therefore$ $\angle C = 40^\circ$ ---- (v)

$\therefore$ $\boldsymbol{\angle F = 40^\circ}$ ---- [From (iv) and (v)]

10. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow of length 40 m on the ground. Determine the height of the tower. **[2 marks]**

Solution:



Let AB represent the vertical stick, AB = 12 m.

BC represents the shadow of the stick, BC = 8 m.

PQ represents the height of the tower.

QR represents the shadow of the tower, QR = 40 m.

$\Delta ABC \sim \Delta PQR$

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ---- [c.s.s.t.]

$\therefore$ $\frac{12}{PQ} = \frac{8}{40}$ ---- [Substituting the given values]

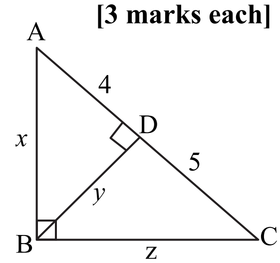
$\therefore$ $PQ = 12 \times 5 = 60$

$\therefore$ **The height of the tower is 60 m.**

---

11. In each of the figures, an altitude is drawn to the hypotenuse. The lengths of different segments are marked in each figure. Determine the value of *x*, *y*, z in each case. **[3 marks each]**

Solution:



i. In ΔABC, $m\angle ABC = 90°$ ---- [Given]

seg BD $\perp$ hypotenuse AC ---- [Given]

$\therefore$ $BD^2 = AD \times DC$ ---- [By property of geometric mean]

$\therefore$ $y^2 = 4 \times 5$ ---- [Substituting the given values]

$\therefore$ $y = \sqrt{4 \times 5}$ ---- [Taking square root on both sides]

$\therefore$ $y = 2\sqrt{5}$ ---- (i)

In ΔADB,

$m\angle ADB = 90°$ ---- [$\because$ Seg BD $\perp$ hypotenuse AC]

$AB^2 = AD^2 + BD^2$ ---- [By Pythagoras theorem]

$\therefore$ $x^2 = (4)^2 + y^2$ ---- [Substituting the given values]

$\therefore$ $x^2 = 4^2 + (2\sqrt{5})^2$ ---- [From (i)]

$\therefore$ $x^2 = 16 + 20$

$\therefore$ $x^2 = 36$

$\therefore$ $x = 6$ ---- [Taking square root on both sides]

In ΔBDC,

$m\angle BDC = 90°$ ---- [$\because$ Seg BD $\perp$ hypotenuse AC]

$\therefore$ $BC^2 = BD^2 + CD^2$ ---- [By Pythagoras theorem]

$\therefore$ $z^2 = y^2 + (5)^2$ ---- [Substituting the given values]

$\therefore$ $z^2 = (2\sqrt{5})^2 + (5)^2$ ---- [From (i)]

$\therefore$ $z^2 = 20 + 25$

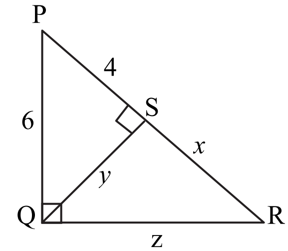
$\therefore$ $z^2 = 45$

$\therefore$ $z = \sqrt{9 \times 5}$ ---- [Taking square root on both sides]

$\therefore$ $z = 3\sqrt{5}$

$\therefore$ **$x = 6$, $y = 2\sqrt{5}$ and $z = 3\sqrt{5}$**

ii. In ΔPSQ,



m $\angle$PSQ = 90° ---- [$\because$ Seg QS $\perp$ hypotenuse PR]

$\therefore$ $PQ^2 = PS^2 + QS^2$ ---- [By Pythagoras theorem]

$\therefore$ $(6)^2 = (4)^2 + y^2$ ---- [Substituting the given values]

$\therefore$ $36 = 16 + y^2$

$\therefore$ $y^2 = 36 - 16$

$\therefore$ $y^2 = 20$

$\therefore$ $y = \sqrt{4 \times 5}$ ---- [Taking square root on both sides]

$\therefore$ $y = 2\sqrt{5}$ ---- (i)

In ΔPQR,

seg QS $\perp$ hypotenuse PR ---- [Given]

$\therefore$ $QS^2 = PS \times SR$ ---- [By the property of geometric mean]

$\therefore$ $y^2 = 4 \times x$ ---- [Substituting the given values]

$\therefore$ $\left(2\sqrt{5}\right)^2 = 4x$ ---- [From (i)]

$\therefore$ $20 = 4x$

$\therefore$ $x = \frac{20}{4}$

$\therefore$ $x = 5$ ---- (ii)

In ΔQSR,

m $\angle$QSR = 90° ---- [$\because$ Seg QS $\perp$ hypotenuse PR]

$\therefore$ $QR^2 = QS^2 + SR^2$ ---- [By Pythagoras theorem]

$\therefore$ $z^2 = y^2 + x^2$ ---- [Substituting the given values]

$\therefore$ $z^2 = (2\sqrt{5})^2 + (5)^2$ ---- [From (i) and (ii)]

$\therefore$ $z^2 = 20 + 25$

$\therefore$ $z^2 = 45$

$\therefore$ $z = \sqrt{9 \times 5}$ ---- [Taking square root on both sides]

$\therefore$ $z = 3\sqrt{5}$

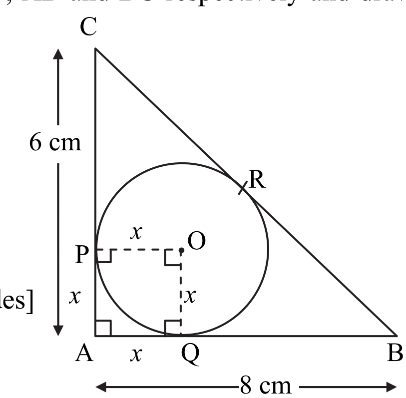
$\therefore$ **$x = 5, y = 2\sqrt{5}$ and $z = 3\sqrt{5}$**

---

12. ΔABC is a right angled triangle with $\angle$A = 90°. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle. [4 marks]

Construction: Let P, Q and R be the points of contact of tangents AC, AB and BC respectively and draw segments OP and OQ.

Solution:



In ΔABC,

$\angle$BAC = 90° ---- [Given]

$\therefore$ $BC^2 = AC^2 + AB^2$ ---- [By Pythagoras theorem]

$\therefore$ $BC^2 = (6)^2 + (8)^2$ ---- [Substituting the given values]

$\therefore$ $BC^2 = 36 + 64$

$\therefore$ $BC^2 = 100$

$\therefore$ BC = 10 units ---- (i) [Taking square root on both sides]

Let the radius of the circle be x cm.

$\therefore$ OP = OQ = x ---- [Radii of same circle]

In □OPAQ,

$\angle$OPA = $\angle$OQA = 90° ---- [Radius is $\perp$ to the tangent]

$\angle$PAQ = 90° ---- [Given]

$\therefore$ $\angle$POQ = 90° ---- [Remaining angle]

$\therefore$ □OPAQ is a rectangle ---- [By definition]

But, OP = OQ ---- [Radii of same circle]

$\therefore$ □OPAQ is a square ---- [A rectangle is a square if its adjacent sides are congruent]

$\therefore$ OP = OQ = QA = AP = x ---- [Sides of a square]

Now, AQ + BQ = AB ---- [A–Q–B]

∴ x + BQ = 8 ---- [Substituting the given values]

∴ BQ = 8 – x

AP + CP = AC ---- [A–P–C]

∴ x + CP = 6 ---- [Substituting the given values]

∴ CP = 6 – x

BQ = BR = 8 – x ---- (ii) } [Length of tangent segments drawn from a external point

CP = CR = 6 – x ---- (iii) } to the circle are equal.]

BC = CR + BR ---- (iv) [C–R–B]

∴ 10 = 6 – x + 8 – x ---- [From (i), (ii), (iii) and (iv)]

∴ $2x = 4$

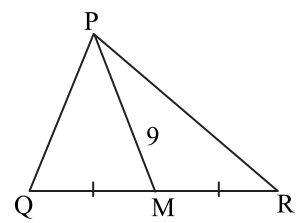
∴ $x = 2$

∴ **The radius of the circle is 2 cm.**

---

13. In ΔPQR, seg PM is a median. If PM = 9 and $PQ^2 + PR^2 = 290$, find QR. **[2 marks]**

Solution:



In ΔPQR,

seg PM is the median ---- [Given]

∴ $PQ^2 + PR^2 = 2PM^2 + 2MR^2$ ---- [By Apollonius theorem]

∴ $290 = 2(9)^2 + 2MR^2$ ---- [Substituting the given values]

∴ $290 = 2(81) + 2MR^2$

∴ $290 = 162 + 2MR^2$

∴ $2MR^2 = 290 - 162$

∴ $2MR^2 = 128$

∴ $MR^2 = \dfrac{128}{2}$

∴ $MR^2 = 64$

∴ MR = 8 ---- (i) [Taking square root on both sides]

Also, QR = 2MR ---- [∵ M is the midpoint of seg QR]

∴ QR = 2 × 8 ---- [From (i)]

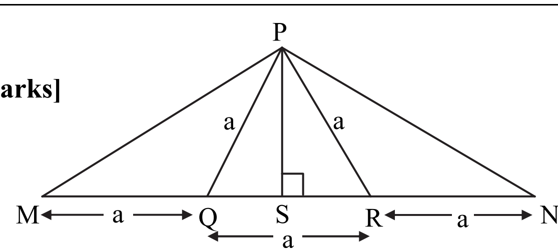
∴ **QR = 16**

---

14. From the information given in the adjoining figure,

Prove that: PM = PN = $\sqrt{3}$ × a, where QR = a. **[4 marks]**

Proof:



In ΔPMR,

QM = QR = a ---- [Given]

∴ Q is midpoint of seg MR.

∴ seg PQ is the median

∴ $PM^2 + PR^2 = 2PQ^2 + 2QM^2$ ---- [By Apollonius theorem]

∴ $PM^2 + a^2 = 2a^2 + 2a^2$ ---- [Substituting the given values]

∴ $PM^2 + a^2 = 4a^2$ ∴ $PM^2 = 4a^2 - a^2$

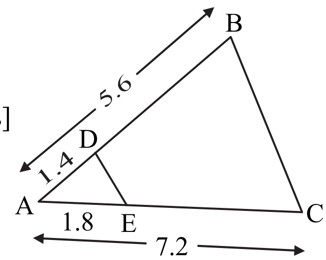
∴ $PM^2 = 3a^2$ ∴ PM = $\sqrt{3}$ a ---- [Taking square root on both sides]

Similarly, we can prove

PN = $\sqrt{3}$ a

∴ **PM = PN = $\sqrt{3}$ a**

15. D and E are the points on sides AB and AC such that AB = 5.6, AD = 1.4, AC = 7.2 and AE = 1.8. Show that DE || BC. [2 marks]



Proof:

DB = AB − AD ---- [A–D–B]

∴ DB = 5.6 − 1.4 ---- [Substituting the given values]

∴ DB = 4.2 units

∴ $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ ---- (i)

Also, EC = AC − AE ---- [A–E–C]

∴ EC = 7.2 − 1.8 ---- [Substituting the given values]

∴ EC = 5.4 units

∴ $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ ---- (ii)

In ΔABC,

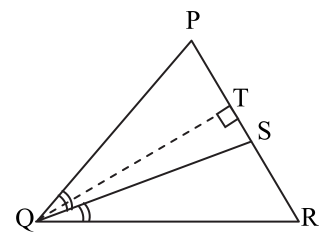
$\frac{AD}{DB} = \frac{AE}{EC}$ ---- [From (i) and (ii)]

∴ **seg DE || seg BC** ---- [By converse of B.P.T.]

---

16. In ΔPQR, if QS is the angle bisector of ∠Q, then show that $\frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{PQ}{QR}$ [3 marks]

(Hint: Draw QT ⊥ PR)



Proof:

In ΔPQR,

Ray QS is the angle bisector of ∠PQR ---- [Given]

∴ $\frac{PQ}{QR} = \frac{PS}{SR}$ ---- (i) [By property of angle bisector of a triangle]

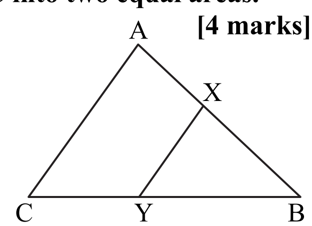
Height of ΔPQS = Height of ΔQRS = QT

∴ $\frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{PS}{SR}$ ---- (ii) [Ratio of areas of two triangles having equal heights is equal to the ratio of their corresponding bases]

∴ $\boldsymbol{\frac{A(\Delta PQS)}{A(\Delta QRS)} = \frac{PQ}{QR}}$ ---- [From (i) and (ii)]

---

17. In the adjoining figure, XY || AC and XY divides the triangular region ABC into two equal areas. Determine AX : AB. [4 marks]



Solution:

seg XY || side AC on transversal BC

∠XYB ≅ ∠ACB ---- (i) [Corresponding angles]

In ΔXYB and ΔACB,

∠XYB ≅ ∠ACB ---- [From (i)]

∠ABC ≅ ∠XBY ---- [Common angle]

∴ ΔXYB ~ ΔACB ---- [By A−A test of similarity]

$\frac{A(\Delta XYB)}{A(\Delta ACB)} = \frac{XB^2}{AB^2}$ ---- (ii) [By theorem on areas of similar triangles]

Now, A(ΔXYB) = $\frac{1}{2}$ A(ΔACB) ---- [∵ seg XY divides the triangular region ABC into two equal areas]

∴ $\frac{A(\Delta XYB)}{A(\Delta ACB)} = \frac{1}{2}$ ---- (iii)

∴ $\frac{XB^2}{AB^2} = \frac{1}{2}$ ---- [From (ii) and (iii)]

∴ $\frac{XB}{AB} = \frac{1}{\sqrt{2}}$ ---- [Taking square root on both sides]

$\therefore$ $1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$ ---- [Subtracting both sides from 1]

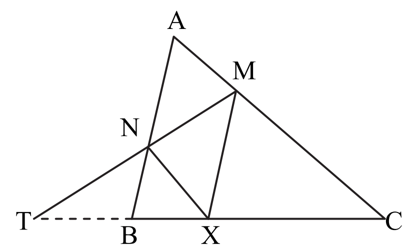
$\therefore$ $\frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$

$\therefore$ $\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$ ---- [A–X–B]

$\therefore$ **$AX : AB = \left(\sqrt{2} - 1\right) : \sqrt{2}$**

---

18. Let X be any point on side BC of ΔABC, XM and XN are drawn parallel to BA and CA. MN meets produced BC in T. Prove that $TX^2 = TB \cdot TC$. [4 marks]



Proof:

In ΔTXM,

seg BN || seg XM ---- [Given]

$\therefore$ $\frac{TN}{NM} = \frac{TB}{BX}$ ---- (i) [By B.P.T.]

In ΔTMC,

seg XN || seg CM ---- [Given]

$\therefore$ $\frac{TN}{NM} = \frac{TX}{CX}$ ---- (ii) [By B.P.T.]

$\therefore$ $\frac{TB}{BX} = \frac{TX}{CX}$ ---- [From (i) and (ii)]

$\therefore$ $\frac{BX}{TB} = \frac{CX}{TX}$ ---- [By invertendo]

$\therefore$ $\frac{BX + TB}{TB} = \frac{CX + TX}{TX}$ ---- [By componendo]

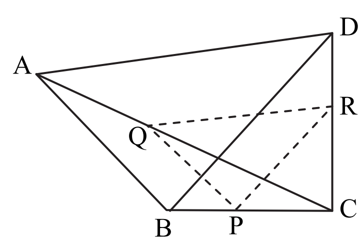
$\therefore$ $\frac{TX}{TB} = \frac{TC}{TX}$ ---- [T–B–X, T–X–C]

$\therefore$ **$TX^2 = TB \cdot TC$**

---

19. Two triangles, ΔABC and ΔDBC, lie on the same side of the base BC. From a point P on BC, PQ || AB and PR || BD are drawn. They intersect AC at Q and DC at R.

Prove that QR || AD. [3 marks]



Proof:

In ΔCAB,

seg PQ || seg AB ---- [Given]

$\therefore$ $\frac{CP}{PB} = \frac{CQ}{AQ}$ ---- (i) [By B.P.T.]

In ΔBCD,

seg PR || seg BD ---- [Given]

$\therefore$ $\frac{CP}{PB} = \frac{CR}{RD}$ ---- (ii) [By B.P.T.]

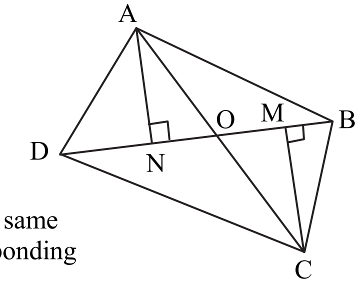
In ΔACD,

$\therefore$ $\frac{CQ}{AQ} = \frac{CR}{RD}$ ---- [From (i) and (ii)]

$\therefore$ **seg QR || seg AD** ---- [By converse of B.P.T.]

20. **In the figure, ΔADB and ΔCDB are on the same base DB. If AC and BD intersect at O, then prove that $\frac{A(\Delta ADB)}{A(\Delta CDB)} = \frac{AO}{CO}$**

[3 marks]



Proof:

$\frac{A(\Delta ADB)}{A(\Delta CDB)} = \frac{AN}{CM}$ ----(i) [Ratio of areas of two triangles with the same base is equal to the ratio of their corresponding heights]

In ΔANO and ΔCMO,

∠ANO ≅ ∠CMO ---- [Each is 90°]

∠AON ≅ ∠COM ---- [Vertically opposite angles]

∴ ΔANO ~ ΔCMO ---- [By A–A test of similarity]

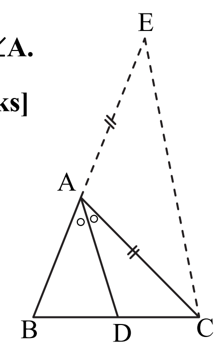
∴ $\frac{AN}{CM} = \frac{AO}{CO}$ ---- (ii) [c.s.s.t.]

∴ $\frac{A(\Delta ADB)}{A(\Delta CDB)} = \frac{AO}{CO}$ ---- [From (i) and (ii)]

---

21. **In ΔABC, D is a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠A.**

(Hint: Produce BA to E such that AE = AC. Join EC) **[5 marks]**



Proof:

seg BA is produced to point E such that AE = AC and seg EC is drawn.

$\frac{BD}{DC} = \frac{AB}{AC}$ ---- (i) [Given]

AC = AE ---- (ii) [By construction]

∴ $\frac{BD}{DC} = \frac{AB}{AE}$ ---- (iii) [Substituting (ii) in (i)]

∴ seg AD || seg EC ---- [By converse of B.P.T.]

On transversal BE,

∠BAD ≅ ∠BEC ---- [Corresponding angles]

∴ ∠BAD ≅ ∠AEC ---- (iv) [∵ B – A – E]

On transversal AC,

∠CAD ≅ ∠ACE ---- (v) [Alternate angles]

In ΔACE,

seg AC ≅ seg AE ---- [By construction]

∠AEC ≅ ∠ACE ---- (vi) [By isosceles triangle theorem]

∴ ∠BAD ≅ ∠CAD ---- [From (iv), (v) and (vi)]

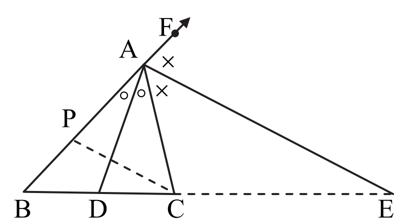
∴ **Ray AD is the bisector of ∠BAC**

---

22. **The bisector of interior ∠A of ΔABC meets BC in D. The bisector of exterior ∠A meets BC produced in E. Prove that $\frac{BD}{BE} = \frac{CD}{CE}$.**

$\left(\text{Hint: For the bisector of } \angle A \text{ which is exterior of } \Delta BAC, \frac{AB}{AC} = \frac{BE}{CE}\right)$

[5 marks]



Construction: Draw seg CP || seg AE meeting AB at point P.

Proof:

In ΔABC,

Ray AD is bisector of ∠BAC ---- [Given]

∴ $\frac{AB}{AC} = \frac{BD}{CD}$ ---- (i) [By property of angle bisector of triangle]

In ΔABE,
seg CP || seg AE ---- [By construction]

$\therefore \frac{BC}{CE} = \frac{BP}{AP}$ ---- [B. P. T]

$\frac{BC+CE}{CE} = \frac{BP+AP}{AP}$ ---- [By componendo]

$\therefore \frac{BE}{CE} = \frac{AB}{AP}$ ---- (ii)

seg CP || seg AE on transversal BF.
$\angle$FAE $\cong$ $\angle$APC ---- (iii) [Corresponding angles]
seg CP || seg AE on transversal AC.
$\angle$CAE $\cong$ $\angle$ACP ---- (iv) [Alternate angles]
Also, $\angle$FAE $\cong$ $\angle$CAE ---- (v) [Seg AE bisects $\angle$FAC]
$\therefore$ $\angle$APC $\cong$ $\angle$ACP ---- (vi) [From (iii), (iv) and (v)]
In ΔAPC,
$\angle$APC $\cong$ $\angle$ACP ---- [From (vi)]
$\therefore$ AP = AC ---- (vii) [By converse of isosceles triangle theorem]

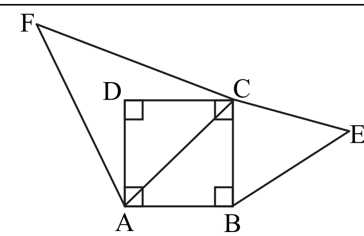
$\therefore \frac{BE}{CE} = \frac{AB}{AC}$ ---- (viii) [From (ii) and (vii)]

$\therefore \frac{BD}{CD} = \frac{BE}{CE}$ ---- [From (i) and (viii)]

$\therefore \mathbf{\frac{BD}{BE} = \frac{CD}{CE}}$ ---- [By alternendo]

---

23. In the adjoining figure, □ABCD is a square. ΔBCE on side BC and ΔACF on the diagonal AC are similar to each other. Then, show that A(ΔBCE) = $\frac{1}{2}$ A(ΔACF). [3 marks]



Proof:
□ABCD is a square. ---- [Given]
$\therefore$ AC = $\sqrt{2}$ BC ---- (i) [$\because$ Diagonal of a square = $\sqrt{2}$ × side of square]
ΔBCE ~ ΔACF ---- [Given]

$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(AC)^2}$ ---- (ii) [By theorem on areas of similar triangles]

$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(\sqrt{2}.BC)^2}$ ---- [From (i) and (ii)]

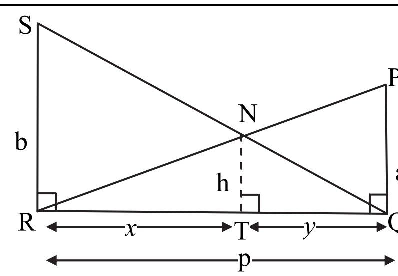
$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{BC^2}{2BC^2}$

$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{1}{2}$

$\therefore$ **A(ΔBCE) = $\frac{1}{2}$ A(ΔACF)**

---

24. Two poles of height 'a' meters and 'b' metres are 'p' meters apart. Prove that the height 'h' drawn from the point of intersection N of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres. [4 marks]



Proof:
Let RT = x and TQ = y.
In ΔPQR and ΔNTR,
$\angle$PQR $\cong$ $\angle$NTR ---- [Each is 90°]
$\angle$PRQ $\cong$ $\angle$NRT ---- [Common angle]

∴ $\Delta PQR \sim \Delta NTR$ ---- [By A – A test of similarity]

∴ $\frac{PQ}{NT} = \frac{QR}{TR}$ ---- [c.s.s.t.]

∴ $\frac{a}{h} = \frac{p}{x}$ ---- [Substituting the given values]

∴ $x = \frac{ph}{a}$ ---- (i)

In ΔSRQ and ΔNTQ,

$\angle SRQ \cong \angle NTQ$ ---- [Each is 90°]

$\angle SQR \cong \angle NQT$ ---- [Common angle]

$\Delta SRQ \sim \Delta NTQ$ ---- [By A–A test of similarity]

∴ $\frac{SR}{NT} = \frac{QR}{QT}$ ---- [c.s.s.t]

∴ $\frac{b}{h} = \frac{p}{y}$ ---- [Substituting the given values]

∴ $y = \frac{ph}{b}$ ---- (ii)

$x + y = \frac{ph}{a} + \frac{ph}{b}$ ---- [Adding (i) and (ii)]

∴ $p = ph\left(\frac{1}{a} + \frac{1}{b}\right)$ ---- [R – T – Q]

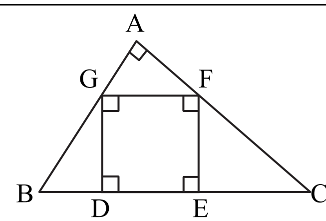
∴ $\frac{p}{ph} = \frac{b+a}{ab}$

∴ $\frac{1}{h} = \frac{a+b}{ab}$

∴ $\mathbf{h = \frac{ab}{a+b}}$ **metres** ---- [By invertendo]

---

25. In the adjoining figure, □DEFG is a square and ∠BAC = 90°.
Prove that: i. ΔAGF ~ ΔDBG ii. ΔAGF ~ ΔEFC
iii. ΔDBG ~ ΔEFC iv. DE^2 = BD·EC
[5 marks]



Proof:

i □DEFG is a square. ---- [Given]

seg GF || seg DE ---- [Opposite sides of a square]

∴ seg GF || seg BC ---- (i) [B–D–E–C]

In ΔAGF and ΔDBG,

∠GAF ≅ ∠BDG ---- [Each is 90°]

∠AGF ≅ ∠DBG ---- [Corresponding angles of parallel lines GF and BC]

∴ **ΔAGF ~ ΔDBG** ---- (ii) [By A–A test of similarity]

---

ii In ΔAGF and ΔEFC,

∠GAF ≅ ∠FEC ---- [Each is 90°]

∠AFG ≅ ∠ECF ---- [Corresponding angles of parallel lines GF and BC]

∴ **ΔAGF ~ ΔEFC** ---- (iii) [By A–A test of similarity]

---

iii. Since, ΔAGF ~ ΔDBG ---- [From (ii)]

and ΔAGF ~ ΔEFC ---- [From (iii)]

∴ **ΔDBG ~ ΔEFC** ---- [From (ii) and (iii)]

iv. Since, $\Delta DBG \sim \Delta EFC$

$\frac{BD}{FE} = \frac{DG}{EC}$ ---- [c.s.s.t.]

$\therefore$ $DG \times FE = BD \times EC$ ---- (iv)

But, $DG = EF = DE$ ---- (v) [Sides of a square]

$\therefore$ $DE \times DE = DB \times EC$ ---- [From (iv) and (v)]

$\therefore$ $\mathbf{DE^2 = BD \cdot EC}$

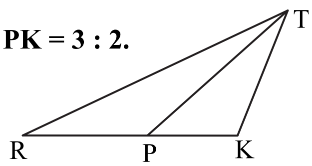
One-Mark Questions

1. In ΔABC and ΔXYZ, $\frac{AB}{YZ} = \frac{BC}{ZX} = \frac{AC}{XY}$, then state by which correspondence are ΔABC and ΔXYZ similar.

Solution:

$\Delta ABC \sim \Delta XYZ$ by $ABC \leftrightarrow YZX$.

2. In the figure, RP : PK = 3 : 2. Find $\frac{A(\Delta TRP)}{A(\Delta TPK)}$.



Solution:

Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.

$\therefore$ $\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{RP}{PK} = \mathbf{\frac{3}{2}}$

3. Write the statement of Basic Proportionality Theorem.

Solution:

If a line parallel to a side of a triangle intersects the other sides in two distinct points, then the line divides those sides in proportion.

4. What is the ratio among the length of the sides of any triangle of angles 30°– 60°– 90°?

Solution:

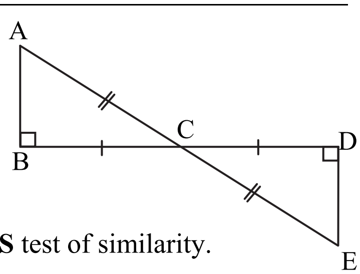
The ratio is $\mathbf{1 : \sqrt{3} : 2}$.

5. What is the ratio among the length of the sides of any triangle of angles 45°– 45°– 90°?

Solution:

The ratio is $\mathbf{1 : 1 : \sqrt{2}}$.

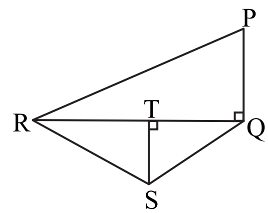
6. State the test by which the given triangles are similar.



Solution:

$\Delta ABC \sim \Delta EDC$ by **SAS** test of similarity.

7. In the adjoining figure, find $\frac{A(\Delta PQR)}{A(\Delta RSQ)}$.



Solution:

Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.

$\therefore$ $\frac{A(\Delta PQR)}{A(\Delta RSQ)} = \mathbf{\frac{PQ}{ST}}$

8. Find the diagonal of a square whose side is 10 cm. [Mar 15]

Solution:

Diagonal of a square $= \sqrt{2} \times$ side.

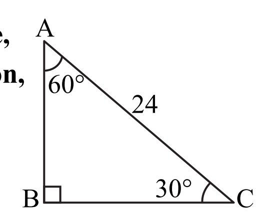
$= \sqrt{2} \times (10) = \mathbf{10\sqrt{2}\ cm}$

9. Adjacent sides of parallelogram are 11 cm and 17 cm respectively. If length of one diagonal is 26 cm, then using which theorem/property can we find the length of the other diagonal?

Solution:

We can find the length of the other diagonal by using Apollonius' theorem.

10. In the adjoining figure, using given information, find BC.



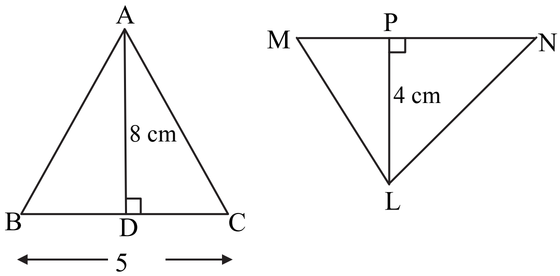
Solution:

$BC = \frac{\sqrt{3}}{2} \times AC$ ---- [Side opposite to 60°]

$= \frac{\sqrt{3}}{2} \times 24$

$\therefore$ $\mathbf{BC = 12\sqrt{3}\ units}$

11. Find the value of MN, so that A(ΔABC) = A(ΔLMN).



Solution:

A(ΔABC) = A(ΔLMN)

$\therefore \quad \frac{1}{2} \times BC \times AD = \frac{1}{2} \times MN \times LP$

$\therefore \quad \frac{1}{2} \times 5 \times 8 = \frac{1}{2} \times MN \times 4$

$\therefore \quad MN = \frac{5 \times 8}{4}$

$\therefore$ **MN = 10 cm**

---

12. If the sides of a triangle are 6 cm, 8 cm and 10 cm respectively, determine whether the triangle is right angled triangle or not. [Mar 14]

Solution:

Note that,

$6^2 + 8^2 = 10^2$,

$\therefore$ By converse of Pythagoras theorem, the given triangle is a right angled triangle.

---

13. Sides of the triangle are 7 cm, 24 cm and 25 cm. Determine whether the triangle is right-angled triangle or not. [Oct 14]

Solution:

The longest side is 25 cm.

$\therefore \quad (25)^2 = 625$ ….(i)

Now, sum of the squares of the other two sides will be

$(7)^2 + (24)^2 = 49 + 576$

$= 625$ ….(ii)

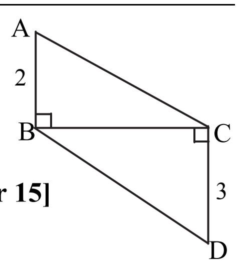
$\therefore \quad (25)^2 = (7)^2 + (24)^2$ ….[From (i) and (ii)]

Yes, the given sides form a right angled triangle.

….[By converse of Pythagoras theorem]

---

14. In the following figure seg AB ⊥ seg BC, seg DC ⊥ seg BC. If AB = 2 and DC = 3, find $\frac{A(\Delta ABC)}{A(\Delta DCB)}$. [Mar 15]



Solution:

Ratio of the areas of two triangles having equal base is equal to the ratio of their corresponding heights.

$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{AB}{DC} \qquad \therefore \quad \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{2}{3}$

---

15. Find the diagonal of a square whose side is 16 cm. [July 15]

Solution:

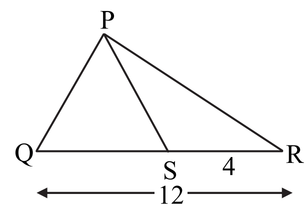
Diagonal of a square $= \sqrt{2} \times$ side.

$= \sqrt{2} \times 16 = \mathbf{16\sqrt{2}}$ **cm**

Additional Problems for Practice

Based on Exercise 1.1

1. In the adjoining figure, QR = 12 and SR = 4. Find values of



i. $\frac{A(\Delta PSR)}{A(\Delta PQR)}$

ii. $\frac{A(\Delta PQS)}{A(\Delta PQR)}$

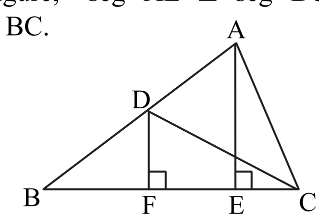
iii. $\frac{A(\Delta PQS)}{A(\Delta PSR)}$ **[3 marks]**

---

2. The ratio of the areas of two triangles with the equal heights is 3 : 4. Base of the smaller triangle is 15 cm. Find the corresponding base of the larger triangle. **[2 marks]**

---

3. In the adjoining figure, seg AE ⊥ seg BC and seg DF ⊥ seg BC. Find



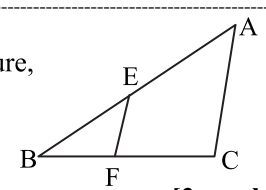
i. $\frac{A(\Delta ABC)}{A(\Delta DBC)}$

ii. $\frac{A(\Delta DBF)}{A(\Delta DFC)}$

iii. $\frac{A(\Delta AEC)}{A(\Delta DBF)}$ **[2 marks]**

Based on Exercise 1.2

4. In the adjoining figure, seg EF || side AC, AB = 18, AE = 10, BF = 4.Find BC.



[3 marks]

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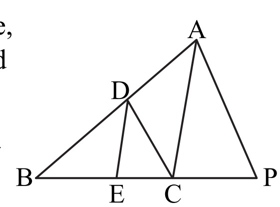
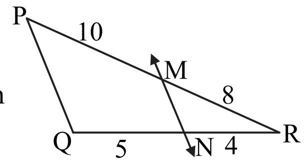
5. In the adjoining figure, seg DE || side AC and seg DC || side AP.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$

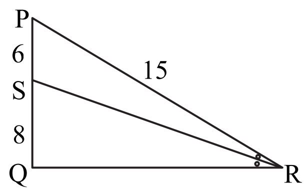


[3 marks]

6. In the adjoining figure, PM = 10, MR = 8, QN = 5, NR = 4. State with reason whether line MN is parallel to side PQ or not? **[2 marks]**

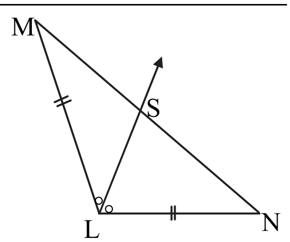


7. In the following figure, in a ΔPQR, seg RS is the bisector of ∠PRQ, PS = 6, SQ = 8, PR = 15. Find QR. **[Mar 15][2 marks]**

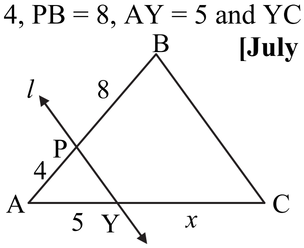


8. Bisectors of ∠B and ∠C in ΔABC meet each other at P. Line AP cuts the side BC at Q. Then prove that $\frac{AP}{PQ} = \frac{AB + AC}{BC}$. **[3 marks]**

9. In the figure given below Ray LS is the bisector of ∠MLN, where seg ML ≅ seg LN, find the relation between MS and SN. **[3 marks]**

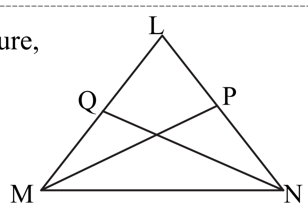


10. In the given figure, line l || side BC, AP = 4, PB = 8, AY = 5 and YC = x. Find x. **[July 15] [2 marks]**

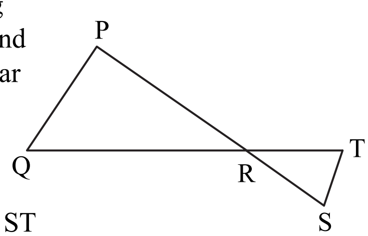


Based on Exercise 1.3

11. In the adjoining figure, ΔMPL ~ ΔNQL, MP = 21, ML = 35, NQ = 18, QL = 24. Find PL and NL. **[2 marks]**



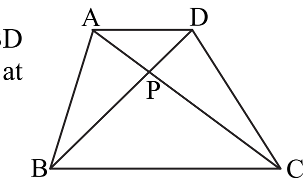
12. In the adjoining figure, ΔPQR and ΔRST are similar under PQR ↔ STR, PQ = 12, PR = 15, $\frac{QR}{TR} = \frac{3}{2}$. Find ST and SR. **[2 marks]**



13. In the map of a triangular field, sides are shown by 8 cm, 7 cm and 6 cm. If the largest side of the triangular field is 400 m, find the remaining sides of the field. **[3 marks]**

14. ΔEFG ~ ΔRST and EF = 8, FG = 10, EG = 6, RS = 4. Find ST and RT. **[2 marks]**

15. In □ABCD, side BC || side AD. Diagonals AC and BD intersect each other at P. If AP = $\frac{1}{3}$AC, then prove that DP = $\frac{1}{2}$BP. **[Oct 09] [4 marks]**



Based on Exercise 1.4

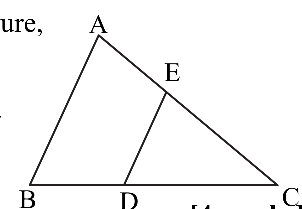
16. If ΔPQR ~ ΔPMN and 9A(ΔPQR) = 6A(ΔPMN), then find $\frac{QR}{MN}$. **[2 marks]**

17. ΔLMN ~ ΔRST and A(ΔLMN) = 100 sq. cm, A(ΔRST) = 144 sq. cm, LM = 5 cm. Find RS. **[2 marks]**

18. ΔABC and ΔDEF are equilateral triangles. A(ΔABC) : A(ΔDEF) = 1 : 2 and AB = 4 cm. Find DE. **[2 marks]**

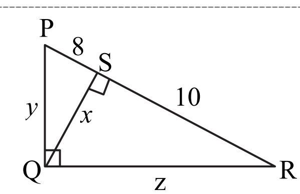
19. If the areas of two similar triangles are equal, then prove that they are congruent. **[4 marks]**

20. In the adjoining figure, seg DE || side AB, DC = 2BD, A(ΔCDE) = 20 cm^2. Find A(□ABDE). **[4 marks]**

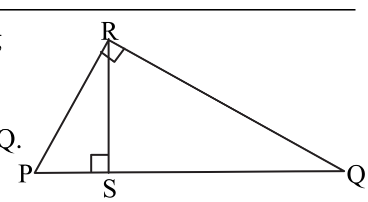


Based on Exercise 1.5

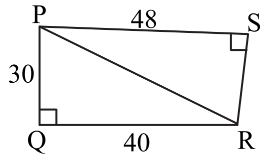
21. In the adjoining figure, ∠PQR = 90°, seg QS ⊥ side PR. Find values of x, y and z. **[3 marks]**



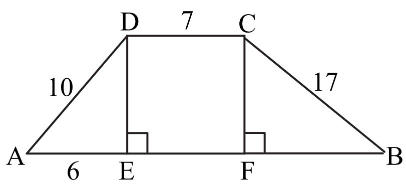
22. In the adjoining figure, ∠PRQ = 90°, seg RS ⊥ seg PQ. Prove that : $\frac{PR^2}{QR^2} = \frac{PS}{QS}$ **[3 marks]**



23. In the adjoining figure,
$\angle PQR = 90°$,
$\angle PSR = 90°$.
Find:
i. PR and ii. RS **[3 marks]**



24. In the adjoining figure,
□ ABCD is a trapezium, seg AB || seg DC,
seg DE ⊥ side AB, seg CF ⊥ side AB.

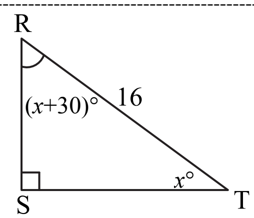


Find: i. DE and CF ii. BF
iii. AB. **[5 marks]**

25. Starting from Anil's house, Peter first goes 50 m to south, then 75 m to west, then 62 m to North and finally 40 m to east and reaches Salim's house. Then find the distance between Anil's house and Salim's house. **[5 marks]**

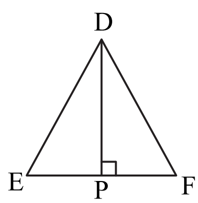
Based on Exercise 1.6

26. In the adjoining figure,
$\angle S = 90°$, $\angle T = x°$,
$\angle R = (x + 30)°$,
RT = 16.
Find: i. RS
ii. ST



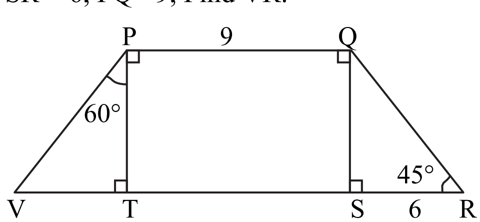
[3 marks]

27. ΔDEF is an equilateral triangle.
seg DP ⊥ side EF,
and E–P–F.
Prove that :
$DP^2 = 3\ EP^2$



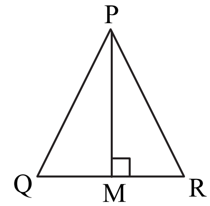
[Oct 08] [4 marks]

28. In the adjoining figure,
□PQRV is a trapezium, seg PQ || seg VR.
SR = 6, PQ= 9, Find VR.



[Mar 13] [3 marks]

29. In the adjoining figure, ΔPQR is an equilateral triangle,
seg PM ⊥ side QR.
Prove that:
$PQ^2 = 4QM^2$



[3 marks]

Based on Exercise 1.7

30. In ΔPQR, seg PM is a median. PM = 10 and $PQ^2 + PR^2 = 362$. Find QR. **[2 marks]**

31. Adjacent sides of a parallelogram are 11 cm and 17 cm. Its one diagonal is 12 cm. Find its other diagonal. **[4 marks]**

32. In ΔABC, $\angle ABC = 90°$, AB = 12, BC = 16 and seg BP is a median. Find BP. **[3 marks]**

Answers to additional problems for practice

1. i. $\frac{1}{3}$ ii. $\frac{2}{3}$ iii. $\frac{2}{1}$
2. 20 cm
3. i. $\frac{AE}{DF}$ ii. $\frac{BF}{FC}$
   iii. $\frac{EC \times AE}{BF \times DF}$
4. 9 units
6. Yes, line MN | | side PQ
7. 20 units
9. seg MS ≅ seg SN
10. 10 unit
11. PL = 28 units and NL = 30 units
12. ST = 8 units and SR = 10 units
13. Remaining sides of field are 350 m and 300 m.
14. ST = 5 units and RT = 3 units
16. $\frac{4}{3}$
17. 6 cm
18. $4\sqrt{2}$ cm
20. 25 cm^2
21. $x = 4\sqrt{5}$ units, $y = 12$ units and $z = 6\sqrt{5}$ units
23. i. 50 units ii. 14 units
24. i. DE = 8 units and CF = 8 units
    ii. BF = 15 units
    iii. AB = 28 units
25. 37 m
26. i. 8 units ii. $8\sqrt{3}$ units
28. $(15 + 6\sqrt{3})$ units
30. 18 units
31. 26 cm
32. 10 units