Model Question Paper – 5

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **$10 \times 1=10$**

1. Give an example of a relation which is reflexive and symmetric but not transitive.
2. Find the value of $cot(\tan^{-1} x + \cot^{-1} x)$.
3. Define a scalar matrix.
4. If $\begin{vmatrix} x & 8 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ 8 & 2 \end{vmatrix}$, find the values of x.
5. Differentiate $\sin\sqrt{x}$ with respect to x.
6. Evaluate: $\int \frac{1-x}{\sqrt{x}} dx$.
7. Find the vector components of the vector with initial point (2,1) and terminal point (-5,7).
8 What is the equation of the plane that cuts the coordinate axes at (a,0,0),(0,b,0) and (0,0,c).
9. Define the term corner point in the L. P. P.
10. If E is an event of a sample space S of an experiment then find P(S/F).

PART B

Answer any TEN questions: **$10 \times 2=20$**

11. Verify whether the operation $*$ defined on Q by $a * b = \frac{ab}{4}$ is associative or not.
12. Find the value of $\tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$.
13. Write the simplest form of $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), 0 < x < \pi$.
14. Let A(1,3), B(0,0) and C(k,0) be the vertices of triangle ABC of area 3 sq. units .Find k using determinant method.
15 Prove that greatest integer function defined by $f(x) = [x],\ 0 < x < 3$ is not differentiable at $x = 1$
16. Find $\frac{dy}{dx}$, If $x = 4t$ and $y = \frac{4}{t}$.
17. Find the intervals in which the function f given by $f(x)=x^2-4x+6$ is strictly increasing.
18. Evaluate: $\int \sin 3x \cos 4x \, dx$.
19. Evaluate: $\int \log x \ dx$.

20. Form the differential equation of the family of curves $\frac{x}{a}+\frac{y}{b}=1$, by eliminating the constants "a" and "b"

21. If either $\vec{a}=\vec{o}$ or $\vec{b}=\vec{o}$ then $\vec{a}\cdot\vec{b}=0$, but the converse need not be true. Justify your answer with an example.

22. Find the angle θ between the vectors $\vec{a}=\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$.

23. Find the distance between the parallel lines

$\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=3\hat{\imath}+3\hat{\jmath}-5\hat{k}+\mu(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$.

24. A fair die is rolled. Consider the events E={1,3,5), F={2,3} and G={2,3,4,5}.

Find (i)P(E/F) (ii) P(E/G)

PART C

Answer any TEN questions: **10×3=30**

25. Determine whether the relation R in the set $A=\{1,2,3,4,5,6\}$ as $R=\{(x,y): y \text{ is divisible by } x\}$ is reflexive, symmetric and transitive.

26. Find the value of x, if $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.

27. Find the values of x and y. $\begin{bmatrix} x+y & 3 \\ x-y & -6 \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 4 & -6 \end{bmatrix}$

28. If $y^x+x^y=a^b$, find $\frac{dy}{dx}$.

29. Verify Mean value theorem for the function $f(x)=x^3-5x^2-3x$, in the interval $[a,b]$ where $a=1$ and $b=3$. Find all $c\in(1,3)$ for which $f'(c)=0$.

30. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

31. Find $\int\frac{x}{(x+1)(x+2)}dx$.

32. Evaluate: $\int\frac{1}{1+\tan x}\,dx$.

33. Find the area of the region bounded by the curve $y=x^2$ and the line $y=4$

34. Prove that the equation $x^2\frac{dy}{dx}=x^2-2y^2+xy$ is a homogeneous differential equation

35. Find a vector perpendicular to each of the vectors $\vec{a}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$,

$\vec{b}=3\hat{\imath}+5\hat{\jmath}-2\hat{k}$,which has magnitude 10 units.

36. Show that the points A(-1,4,-3), B(3,2,-5) C(-3,8,-5) and D(-3,2,1) are coplanar.

37. Find the Cartesian and vector equation of the line that passes through the points (3,-2,-5) and (3,-2,6)

38. A die is thrown. If E is the event "the number appearing is a multiple of 3" and F be the event " The number appearing is even". Then find whether E and F are independent?

PART D

Answer any SIX questions: **6 × 5=30**

39. If $f : A \rightarrow A$ defined by $f(x) = \frac{4x+3}{6x-4}$, where $A = R - \left\{\frac{2}{3}\right\}$,show that f is invertible and $f^{-1} = f$.

40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & -3 & 0 \\ 4 & 5 & -3 \end{bmatrix}$ $and\ C = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 6 \\ -1 & 2 & 3 \end{bmatrix}$, prove that A(B+C)=AB+AC.

41. Solve the following system of equations by matrix method:

    x +2y +3 z = 2 ; 2x + 3y +z = -1 and x - y - z = -2.

42. If $= (\tan^{-1} x)^2$, prove that $(x^2 + 1)^2 y_2 + 2x\,(x^2 + 1)y_1 = 2$

43. A particle moves along the curve 6y=x²+2. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x- coordinate.

44. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ with respect to x, and hence evaluate $\int \frac{dx}{\sqrt{5-4x-x^2}}$

45. Find the area of the of the circle $x^2 + y^2 = a^2$ by the method of the integration and hence find the area of the circle $x^2 + y^2 = 2$.

46. Find the general solution of the differential equation $\frac{dy}{dx} + y \cdot cotx = 2x + x^2 \cdot cotx$.

47. Derive the formula to find the shortest distance between the two skew lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}\ and\ \vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ in the vector form.

48. A fair coin is tossed 8 times. Find the probability of at most 5 Heads.

PART E

Answer any ONE question: **1 × 10=10**

49. (a) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day? 6

(b) Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$=(a-b)(b-c)(c-a). 4

50. (a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$. 6

(b) Find all the points of discontinuity on f defined by $f(x) = |x| - |x+1|$. 4