Model Question Paper – 2

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

Answer ALL the questions **10×1=10**

1. Give an example of a relation which is symmetric only.
2. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.
3. Construct a 2x2 matrix $A=\left[a_{ij}\right]$.Whose element are given by a_{ij}=2i+j .
4. If A=$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, find $|2A|$.
5. If y=$e^{3\log x}$, then show that $\frac{dy}{dx} = 3x^2$.
6. Find the antiderivative of $x^2\left(1-\frac{1}{x^2}\right)$ with respect to x.
7. Define the feasible region in LPP.
8. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.
9. Find the direction ratio of the line $\frac{x-1}{2} = 3y = \frac{2z+3}{4}$.
10. If P(E)=0.6 and P(E∩F)=0.2 then find P(F/E).

PART B

Answer any TEN questions: **10×2=20**

11. Define binary operation on a set. Verify whether the operation * defined on Z by a*b=ab+1 is binary or not.
12. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ in the simplest form.
13. Find the equation of line passing through the points (3,2) and (-1,-3) by using determinants.
14. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x, \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$.
15. If y= $\tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right], \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ then find $\frac{dy}{dx}$.

16. Find $\frac{dy}{dx}$, if $\sin^2 x + cos^2 y = k$, where k is constant.

17. If the radius of a sphere is measured as 7cm with an error of 0.02m, then find the approximate error in calculating its volume.

18. Evaluate: $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\, dx$.

19. Evaluate: $\int \tan^{-1} x\, dx$.

20. Find the projection of the vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$.

21. Find the area of parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$.

22. Find the distance of a point (2,5,-7) from the plane $\vec{r}\cdot(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.

23. Find the order and the degree of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

24. Given that the event A and B are such that P(A)= $\frac{1}{2}$, P(A∩B)=$\frac{3}{5}$ and P(B)=k, find k if A and B are independent.

PART C

Answer any TEN questions: **10×3=30**

25. Show that the relation R in the set of all integers Z defined by R= {(a, b): 2 divides a-b} is an equivalence relation.

26. Simplify: $\tan^{-1}\left[\frac{2cosx-3sinx}{3cosx+2sinx}\right]$, $\frac{2}{3}tanx > -1$

27. Express matrix A= $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

28. Prove that if the function is differentiable at a point c,then it is also continuous at that point.

29. Verify mean value theorem for the function f(x)=x^2-4x-3 in the interval [1,4].

30. Find the equation of tangent to the curve given by x=a$\sin^3$t ,y=b$\cos^3$t at a point where t= $\frac{\pi}{2}$.

31. Evaluate: $\int \frac{x+2}{2x^2+6x+5}\, dx$.

32. Evaluate: $\int e^x \left(\frac{1+sinx}{1+cosx}\right) dx$.

33. Find the area bounded by parabola y^2=4x and the line y=2x.

34. Three vectors $\vec{a}, \vec{b}$ & $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$.

    Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$, if $|\vec{a}| = 1, |\vec{b}| = 4$ & $|\vec{c}| = 2$.

35. If $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$ then find $\vec{a} \cdot (\vec{b} \times \vec{c})$ and $(\vec{a} \times \vec{b}) \cdot \vec{c}$

36. Find the shortest distance between the lines $l_1: \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $l_2 : \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

37. Form the differential equation of the family of circle touching the x-axis at origin.

38. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 trucj drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

PART D

Answer any SIX questions: **6×5=30**

39. Let $f: N \to R$ defined by$f(x) = 4x^2 + 12x + 15$, show that f : $N \to S$, where S is the range of the function is invertible. Also find the inverse of f.

40. Verify (B+C)A =BA+ CA, if A= $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 8 \\ 11 & 21 \end{bmatrix}$ and C = $\begin{bmatrix} 7 & 13 \\ 5 & 19 \end{bmatrix}$

41. Solve the equations x-y+3z=10, x-y-z=-2 and 2x+3y+4z=4 by matrix method.

42. If $y = Ae^{mx} + Be^{nx}$ then prove that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

43. A ladder $5\,m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of $2\,cm/sec$. How fast is its height on the wall decreasing when the foot of the ladder is $4\,m$ away from the wall?

44. Find the integral of $\frac{1}{\sqrt{x^2-a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2 - 2x}} dx$.

45. Using integration find the area bounded by the circle $x^2 + y^2 = 16$ and the parabola $y^2=6x$.

46. Derive the equation of the line in space passing through a point and parallel to a vector both in the vector and Cartesian form.

47. Solve the differential equation $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$.

48. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

PART E

Answer any ONE question: **1×10=10**

49. (a) One kind of cake requires 200g of flour and 25g of fat and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredient, used in making the cakes.

(b) Find the value of K if $f(x)=\begin{cases} Kx+1 & if\ x\le 5 \\ 3x-5 & if\ x>5 \end{cases}$ is continuous at x = 5.

50. (a) Prove that

$$\int_{-a}^{a} f(x)dx = \begin{cases} \int_0^{2a} f(x)dx\,, & if\ f(x)\ is\ even \\ 0 \qquad\qquad , & if\ f(x)\ is\ odd \end{cases}$$

and hence evaluate $\int_{-\pi/2}^{\pi/2} \tan^9 x\, dx$.

(b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$=ab+bc+ca+abc