Model Question Paper – 1

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART-A

Answer All the questions: **10×1 = 10**

1. Operation * is defined by a*b=a. Is * is a binary operation on Z^+?
2. Write the principal value branch of $f(x) = \sin^{-1}x$.
3. Define a diagonal matrix.
4. If $A = \begin{pmatrix} 4 & 7 \\ 6 & 5 \end{pmatrix}$ find $|3A|$.
5. Write the points of discontinuity for the function $f(x) = [x], \ -3 < x < 3$.
6. Evaluate $\int \operatorname{cosec} x(\operatorname{cosec} x - \cot x)dx$
7. Find the direction ratios of the vector, joining the points $P(2,3,0)$ and $Q(-1,-2,-3)$, direction from P to Q.
8. Find the equation of the plane with the intercept 2, 3 and 4 on x, y and z axes respectively.
9. Define optimal solution in the linear programming.
10. If A and b are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, find $P(A \cap B)$.

PART-B

Answer any Ten questions: **10×2 = 20**

11. Find the gof and fog if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$
12. Write the function $tan^{-1}\left(\frac{casx - sinx}{cosx + sinx}\right), \ 0 < x < \pi$ in the simplest form.
13. Prove that $2sin^{-1}\left(\frac{3}{5}\right) = tan^{-1}\left(\frac{24}{7}\right)$.
14. Find the area of a triangle whose vertices are $(1,3), (2,5)$and$(7,5)$ using determinant.
15. Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$.
16. If $x = at^2, \ y = 2at$ show that $\frac{dy}{dx} = \frac{1}{t}$
17. Find the approximate change in the volume V of a cube of a side x meters caused byincreasing by 2 %.

18. Evaluate $\int sin^3xdx$.

19. Evaluate $\int_0^{\frac{\pi}{2}} cos2xdx$.

20. Find the order and degree of differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$.

21. Find a vector in the direction of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ that has magnitude 7 units.

22. If $\vec{a} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

23. Find the vector equation of the line, passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$

24. Two coins are tossed once, find$P(E/F)$ where E: no tail appears, F: no head appears.

PART-C

Answer any Ten questions: **10×3=30**

25. Show that the relation R in the set of real numbers $\boldsymbol{R}$ defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexivenor symmetric nor transitive

26. Prove that $tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), |x| < \frac{1}{3}$

27. Find the values of x, y and z in the following matrices

$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$$

28. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$ with respect to x.

29. If $y = sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$find $\frac{dy}{dx}$

30. If $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) Strictly increasing (b) Strictly decreasing.

31. Evaluate $\int \frac{(1+logx)^2}{x} dx$

32. Evaluate $\int \frac{xsin^{-1}x}{\sqrt{1-x^2}} dx$

33. Find the area between the curves $y = x^2$ and $y = x$

34. Form the differential equation representing the family of curves $y = asin(x + b)$ where a and b arbitrary constants.

35. Find the area of a triangle having the points $A(1, 1, 1), B(1, 2, 3)$and$C(2, 3, 1)$ as its vertices using vector method.

36. Prove that $[\vec{a}, \vec{b}, \vec{c} + \vec{d}] = [\vec{a}, \vec{b}, \vec{c}] + [\vec{a}, \vec{b}, \vec{d}]$.

37. Find the distance between parallel lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + m(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + n(2\hat{i} + 3\hat{j} + 6\hat{k})$

38. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

PART-D

Answer any Six questions: **6×5 = 30**

39. Let $f: N \rightarrow R$ be a function defined by $f(x) = 4x^2 + 12x + 5$. Show that $f: N \rightarrow S$, where S is the range of function f, is invertible. Find the inverse of f.

40. Verify $(A + B)C = AC + BC$,

if $A = \begin{pmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$

41. Solve the following system of equations by matrix method $x + y + z = 6$, $x - y - z = -4$ and $x + 2y - 2z = -1$.

42. If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$ and hence prove that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

43. The length of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When x = 10 cm and y = 6 cm, find the rate of change of (i) the perimeter and (ii) the area of the rectangle

44. Find the integral of $\sqrt{x^2 + a^2}$ w.r.t x and hence evaluate $\int \sqrt{x^2 + 4x + 6}\, dx$

45. Using integration find the area bounded by the triangle whose vertices are $A(2, 0), B(4, 5)$ and $C(6, 3)$

46. Solve the differential equation $ydx - (x + 2y^2)dy = 0\backslash$

47. Derive equation of plane perpendicular to a given vector and passing through a given point both in the vector and Cartesian form.

48. There are 5 % defective items in a large bulk of items. What is the probability that sample of 10items will include not more than one defective item?

PART-E

Answer any One question: **1× 10 = 10**

49. (a) Prove that $\int_0^{2a} f(x)\, dx = 2\int_0^a f(x)\, dx$ when $f(2a - x) = f(x)$ and hence evaluate $\int_0^{\pi} |cosx|\, dx$ 6

(b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (x^3 - 1)^2$ 4

50 (a) Solve the following linear programming problem graphically:

Maximize, $z = 3x + 2y$ subjected to the constraints:

$x + 2y \leq 10,\ 3x + y \leq 15,\ x \geq 0,\ y \geq 0.$ 6

(b) Find the relationship between a and b so that the function f defined by

$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at x = 3. 4