Total No. of Printed Pages—7

HS/XII/A. Sc. Com/M/13

2 0 1 3

MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Find the domain and range of the relation R defined on W by $R \quad \{(a, b) : a, b \quad W$ and $a \quad 3b \quad 12\}$.

2. Show that the operation on Q-$\{1\}$ defined by $a \quad b \quad a \quad b \quad ab$, for all $a, b \quad Q$-$\{1\}$, satisfies the commutative law.

3. Apply the properties of determinant to show that

$$\begin{vmatrix} 24 & 25 & 26 \\ 27 & 28 & 29 \\ 30 & 31 & 32 \end{vmatrix} \quad 0$$

4. Given $X \quad \begin{matrix} 2 & 0 & 2 \\ 1 & 0 & 1 \end{matrix}$ and $Y \quad \begin{matrix} 3 & 1 & 0 \\ 2 & 0 & 1 \end{matrix}$, find Z, such that $X \quad Y \quad Z \quad 0$.

5. Discuss the continuity of the function $f(x)$ at $x \quad 5$, if

$$f(x) \quad \begin{matrix} \frac{x^2 \;\; 25}{x \;\; 5}, & \text{if } x \quad 5 \\ 10 \quad\;\; , & \text{if } x \quad 5 \end{matrix}$$

6. Find dy/dx, if $y \quad \tan(\sin^{\;\;1} x)$.

7. Integrate :

$$x \cos x \, dx$$

8. Show that

$$ {}_{0}^{1} x(1 \quad x)^5 dx \quad \frac{1}{42}$$

9. Find $\frac{dy}{dx}$, when $x \quad a(t \quad \sin t)$ and $y \quad a(1 \quad \cos t)$.

10. Find two positive numbers whose product is 49 and their sum is minimum.

11. Find the angle between the vectors $\vec{a} \quad 3\hat{i} \quad 2\hat{j} \quad \hat{k}$ and $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$.

12. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$ and $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$.

13. If A and B are independent events such that $P(A) \quad 0 \ 3$, $P(B) \quad 0 \ 4$, then find $P(A$ and $B)$.

14. Show that the lines

$$\frac{x \quad 3}{2} \quad \frac{y \quad 1}{3} \quad \frac{z \quad 2}{4} \text{ and } \frac{x \quad 2}{2} \quad \frac{y \quad 4}{4} \quad \frac{z \quad 5}{2}$$

are perpendicular to each other.

15. Let A and B be events such that $P(A) \quad \frac{1}{6}$, $P(B) \quad \frac{1}{4}$ and $P(A \quad B) \quad \frac{1}{12}$. Find $P(B/A)$ $P(A/B)$.

SECTION—B

16. Prove that

$$\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{36}{85}$$

17. Using the properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a - b)(b - c)(c - a)$$

18. The volume of a cube is increasing at the rate of 7 cubic centimeters per second. How fast is its surface area increasing at the instant when the length of an edge of the cube is 12 cm?

19. State the Lagrange's mean value theorem and verify the theorem for the following function :

$$f(x) = 2x - x^2 \text{ in } [0, 1]$$

Or

Find the intervals on which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is *(i)* strictly increasing and *(ii)* strictly decreasing.

20. Solve :

$$(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$$

21. If $y \quad a\cos(\log x) \quad b\sin(\log x)$, then prove that

$$x^2 y_2 \quad x y_1 \quad y \quad 0$$

22. Integrate :

$$\frac{(x \quad 1)e^x}{\cos^2(xe^x)}\,dx$$

23. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \quad (\hat{i} \quad \hat{j} \quad \hat{k}) \quad 6$ and $\vec{r} \quad (2\hat{i} \quad 3\hat{j} \quad 4\hat{k}) \quad\quad 5$ at the point (1, 1, 1).

Or

Find the equation of the plane passing through the points $P(1, \quad 1, 2)$ and $Q(2, \quad 2, 2)$ and perpendicular to the plane $6x \quad 2y \quad 2z \quad 9$.

24. Find the shortest distance between the lines

$$\vec{r} \quad (\hat{i} \quad 2\hat{j} \quad \hat{k}) \quad\quad (\hat{i} \quad \hat{j} \quad \hat{k}) \text{ and}$$
$$\vec{r} \quad (2\hat{i} \quad \hat{j} \quad \hat{k}) \quad\quad (2\hat{i} \quad \hat{j} \quad 2\hat{k})$$

25. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0·01 and that of a motorcycle is 0·02. An insured vehicle met with an accident. Find the probability that the particular vehicle was a motorcycle.

SECTION—C

26. Find the area of the region enclosed by the X-axis, the line $y \quad x$ and the circle $x^2 \quad y^2 \quad 16$ in the first quadrant.

27. Solve the following differential equations :

(i) $(1 \quad x)y\,dx \quad (1 \quad y)x\,dy \quad 0$, when $y(1) \quad 1$

(ii) $(1 \quad x^2)\dfrac{dy}{dx} \quad 2xy \quad \cos x$

28. Solve the following system of equations by matrix method :

$$2x \quad 3y \quad 5z \quad 1$$
$$3x \quad 2y \quad 4z \quad 5$$
$$x \quad y \quad 2z \quad 3$$

Or

If $A \quad \begin{matrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{matrix}$, then verify that

$$A \;(\mathrm{adj}\,A) \quad (\mathrm{adj}\,A)\; A \quad |A|I$$

29. *(a)* If $y \quad (\sin x)^{\tan x}$, then find $\dfrac{dy}{dx}$.

(b) If $y \quad \tan^{\;1} \dfrac{\sqrt{1 \quad x^2} \quad \sqrt{1 \quad x^2}}{\sqrt{1 \quad x^2} \quad \sqrt{1 \quad x^2}}$, then prove that

$$\frac{dy}{dx} \sqrt{1 \quad x^4} \quad x$$

30. A firm manufactures two types of products *A* and *B*, and sells them at a profit of Rs 5 per unit of type *A* and Rs 3 per unit of type *B*. Each product is processed on two machines M_1 and M_2. One unit of type *A* requires one minute of processing time on M_1 and two minutes of processing time on M_2; whereas one unit of type *B* requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

Or

A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine Rs 240. He expects to gain Rs 22 on a fan and Rs 18 on a sewing machine. Assuming that he can sell all the items he buys. How should he invest the money in order to maximize the profit?

★★★