Total No. of Printed Pages—11

HS/XII/A. Sc. Com/M/15

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MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of four Sections—A, B, C and D.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 5 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C consists of 10 questions, carrying *4* marks each.

Question Nos. **21** to **25** are to be answered by both Regular and Private Candidates. Question Nos. **26** to **30** are to be answered by Elementary School Teacher Candidates only.

N. B. : Regular and Private Candidates are not to attempt Question Nos. **26** to **30**.

Elementary School Teacher Candidates are not to attempt Question Nos. **21** to **25**.

(vi) Section—D has 5 questions carrying *6* marks each, out of which two questions have internal choices.

SECTION—A

1. *(a)* If R $\{(1, 1), (2, 2), (3, 1)\}$ is a relation, then find the range of R. 1

(b) If f is a function from $\mathbb{R}$ $\mathbb{R}$ such that $f(x)$ x^2 x $\mathbb{R}$, then show that f is not one-one. 1

2. *(a)* If $\bar{a}$ $\bar{b}$ $\bar{c}$ for a non-zero scalar and non-zero vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$, then find $\bar{a}$ $\bar{c}$. 1

(b) Obtain the dot product of the vectors $\bar{a}$ $\hat{i}$ $\hat{j}$ $\hat{k}$ and $\bar{b}$ $\hat{i}$ $\hat{k}$. 1

3. *(a)* Find the value of $\frac{d}{dx}(\sin^2 x^2 \quad \cos^2 x^2)^4$. 1

(b) What is the value of $\int_1^2 \frac{dx}{x^2}$? 1

4. *(a)* Evaluate :

$$\begin{vmatrix} 2 & 3 & 5 \\ 261 & 592 & 127 \\ 20 & 30 & 50 \end{vmatrix}$$ 1

(b) Compute AB, if A $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ and B $\begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}$. 1

5. By applying properties of determinant, show that

$$\begin{vmatrix} a & b & b & c & c & a \\ b & c & c & a & a & b \\ c & a & a & b & b & c \end{vmatrix} \quad 0$$

6. Find a unit vector perpendicular to both $\bar{a}$ and $\bar{b}$, where $\bar{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$ and $\bar{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$.

7. Find the angle between the lines

$$\frac{5 \quad x}{3} \quad \frac{y \quad 3}{4}, \; z \quad 7$$

and

$$x \quad \frac{1 \quad y}{2} \quad \frac{z \quad 6}{2}$$

8. Find the identity element in $\mathbb{Z}$ with respect to the operation ' ' defined by $a \quad b \quad a \quad b \quad 1 \quad a, b \quad \mathbb{Z}$.

9. Find $\frac{dy}{dx}$, where $x \quad t^3 \quad \frac{1}{t}$ and $y \quad (t \quad t^2)^3$.

10. Solve the differential equation

$$\frac{dy}{dx} \quad \sqrt{4 \quad y^2}$$

11. Find :

$$\log x \, dx$$

12. Evaluate :

$$\int_0^1 x(1 \quad x)^5 dx$$

13. If $y \quad \sqrt{x} \quad \frac{1}{\sqrt{x}}$, then show that

$$2x\frac{dy}{dx} \quad y \quad 2\sqrt{x}$$

14. Amit and Nisha appear for an interview for two vacancies in a company. The probability of Amit's selection is $\frac{1}{5}$ and that of Nisha's selection is $\frac{1}{6}$. What is the probability that both of them are selected?

15. Show that the differential equation of the family of circles having their centre at the origin and radius a is

$$x \quad y\frac{dy}{dx} \quad 0$$

SECTION—B

16. Prove that

$$\cos^{1}\frac{4}{5} \quad \tan^{1}\frac{3}{5} \quad \tan^{1}\frac{27}{11}$$

17. By using differentials, find the approximate value of the cube root of 127 up to 3 places of decimal.

Or

A particle moves along the curve $6y \quad x^3 \quad 2$. Find the points on the curve at which the y coordinate is changing 8 times as fast as the x coordinate.

18. Find the equations of the line passing through the point (1, 2, 4) and perpendicular to each of the lines

$$\frac{x\ 8}{8}\quad\frac{y\ 19}{16}\quad\frac{z\ 10}{7}$$

and

$$\frac{x\ 15}{3}\quad\frac{y\ 29}{8}\quad\frac{z\ 5}{5}$$

19. *(a)* For what value of x, the matrix

$$\begin{matrix} 3 & 2x & x & 1 \\ & 2 & & 4 \end{matrix}$$

is singular? 2

(b) Let

$$A = \begin{matrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{matrix}$$

Find the matrix $\frac{1}{2}(A \quad A)$.

(A is the transpose of A) 2

20. Using Lagrange's mean value theorem, find a point on the parabola $y \quad (x \quad 3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

Or

Find the equation of the tangent to the curve $y \quad \sqrt{5x \quad 3}$ which is parallel to the line $4x \quad 2y \quad 3 \quad 0$.

SECTION—C

21. Evaluate :

$$\sin^4 x\,dx$$

22. Solve the differential equation

$$\frac{dy}{dx} \quad 3y\cot x \quad \sin 2x$$

it being given that $y \quad 2$, when $x \quad / 2$.

23. Show that the lines

$$\bar{r} \quad (\hat{i} \quad \hat{j}) \quad (2\hat{i} \quad \hat{k})$$

and $$\bar{r} \quad (2\hat{i} \quad \hat{j}) \quad (\hat{i} \quad \hat{j} \quad \hat{k})$$

do not intersect.

24. Find the intervals on which the function $f(x) \quad 2x^3 \quad 3x^2 \quad 36x \quad 7$ is *(a)* strictly increasing and *(b)* strictly decreasing.

25. Prove that

$$0 \frac{x\sin x}{1 \quad \cos^2 x}dx \quad \frac{2}{4}$$

(Only for Elementary School Teacher Candidates in lieu of Question Nos. 21 to 25)

26. State if the following are True or False : 1×4=4

(a) $\frac{dy}{dx}$ $5y$ e^{x} is not a differential equation.

(b) The derivative of a constant function is zero.

(c) Every differentiable function is continuous.

(d) k $f(x)\,dx$ $f(x)\,dx$ k.

27. Answer the following : 1×4=4

(a) Find $\frac{dy}{dx}$, if y e^{4x}.

(b) Evaluate $(3x$ $4)\,dx$.

(c) Find $f(2)$, if $f(x)$ 2.

(d) Find the value of $\int_{1}^{3} dx$.

28. Answer the following : 1×4=4

(a) If $\bar{a} \quad 2\hat{i} \quad 3\hat{j} \quad 4\hat{k}$ and $\bar{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$, then find $\bar{a} \quad \bar{b}$.

(b) Show that the vectors $\bar{a} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$ and $\bar{b} \quad \hat{i} \quad 9\hat{j} \quad 3\hat{k}$ are perpendicular.

(c) Find the direction cosines of the vector $2\hat{i} \quad \hat{j} \quad 2\hat{k}$.

(d) If $|\bar{a}| \quad 2, |\bar{b}| \quad 7$ and $\bar{a} \quad \bar{b} \quad 7$, then find the angle between $\bar{a}$ and $\bar{b}$.

29. Choose and write the correct answer : 1×4=4

(a) The value of $2^x dx$ is

(i) $\dfrac{2^{x \quad 1}}{x \quad 1} \quad c$

(ii) $2^x \log 2 \quad c$

(iii) $\dfrac{2^x}{\log 2} \quad c$

(iv) None of the above

(b) The derivative of $\tan x$ w.r.t. x is

(i) $\cot x$

(ii) $\sec^2 x$

(iii) $\sec^2 x$

(iv) $\sec x \tan x$

(c) The order and the degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 \quad 2\left(\frac{dy}{dx}\right)^5 \quad 9y \quad \sin x$$

are

(i) 2, 5

(ii) 1, 5

(iii) 2, 1

(iv) 2, 3

(d) The second-order derivative of $\log x$ with respect to x is

(i) $\frac{1}{x}$

(ii) $\frac{1}{x^2}$

(iii) 1

(iv) $\frac{1}{x^2}$

30. Fill in the blanks with correct answer : 1×4=4

(a) The derivative of $\sin x$ with respect to x is ——.

(b) The value of $e^x dx$ is ——.

(c) $\frac{d}{dx}(\sqrt{x})$ ——.

(d) $3\,dx$ ——.

SECTION—D

31. Find the area of the region

$$\{(x, y): x^2 \quad y^2 \quad 1 \quad x \quad y\}$$

32. The sum of three numbers is 6. Twice the third number when added to the first number gives 7. On adding the sum of the second and the third numbers to thrice the first number, we get 12. Find the numbers by using matrix method.

33. There are 5 cards numbered 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let x denotes the sum of the numbers on the two cards drawn. Find the mean and variance of x.

Or

A company manufactures scooters at two plants A and B. Plant A produces 80% and plant B produces 20% of the total product. 85% of the scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is selected at random and is found to be of standard quality. What is the probability that it was manufactured at plant A?

34. *(a)* If $x^y \quad y^x \quad 1$, then find $\frac{dy}{dx}$. 4

(b) If $y \quad \tan^{1}(\cot x) \quad \cot^{1}(\tan x)$, then prove that

$$\frac{dy}{dx} \quad 2 \quad 0$$ 2

35. A manufacturer produces two types of soap bar using two machines, *A* and *B*. *A* is operated for 2 minutes and *B* for 3 minutes to manufacture the first type, while it take 3 minutes on machine *A* and 5 minutes on machine *B* to manufacture the second type. Each machine can be operated at the most for 8 hours per day. The two types of soap bar are sold at a profit of ₹ 0·25 and ₹ 0·50 each. How many bars of soap of each type should be manufactured per day so as to maximize the profit?

Or

If a young man drives his car at 40 km per hour, he has to spend ₹ 5 per km on petrol; if he drives it at a slower speed of 25 km per hour, the petrol cost decreases to ₹ 2 per km. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.

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