Model Question Paper – 4

II P.U.C MATHEMATICS (35)

Time : 3 hours 15 minute **Max. Marks : 100**

Instructions :

(i) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*

(ii) *Use the graph sheet for the question on Linear programming in PART E.*

PART – A

ANSWER ALL THE QUESTIONS **10X1=10**

1. State with reason whether the function h:$\{2,3,4,5\} \rightarrow \{7,9,11,13\}$ has inverse with $h = \{(2,7),(3,9),(4,11),(5,13)\}$.

2. Find $|3A|$ if A=$\begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$

3. Find the value of $\sin^{-1}(\sin\frac{3\pi}{5})$

4. The greatest integer function a not differentiable at integral points give reason.

5. Construct a 2X2 matrix A=$\left[a_{ij}\right]$, where $a_{ij} = \frac{i-j}{2}$.

6. Evaluate $\int \sin(2+5x)dx$

7. If $\vec{a}$ is a non zero vector of magnitude a and $\lambda\vec{a}$ is a unit vector, find the value of λ.

8. Find the distance of the plane 2x-3y+4z-6=0 from the orgin.

9. Define the term constraints in the LPP.

10.

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

Given is not a probability distribution why?

PART B

Answer any TEN questions: **10 $\times$ 2=20**

11. A binary operation $\wedge$ on the set $\{1,2,3,4,5\}$ defined by $a \wedge b = \min\{a,b\}$, write the operation table for operation $\wedge$.

12. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{7}$

13. Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

14. Find the area of triangle whose vertices are (2,0), (-1,0) and (0,3) by using determinant.

15. Differentiate $x^{\sin x}, x > 0$ w.r.t x

16. If y= $\tan^{-1}\frac{\sin x}{1+\cos x}$ then prove that $\frac{dy}{dx} = \frac{1}{2}$

17. Find the local maximum value of the function g(x)=x^3-3x.

18. Evaluate $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

19. Evaluate $\int \frac{dx}{x^2 - 6x + 13}$

20. Find the order and degree of differential equation: $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$

21. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

22. Find $|\vec{a}|$ *and* $|\vec{b}|$ if $(\vec{a} + \vec{b}).(\vec{a} - \vec{b})$ and $|\vec{a}| = 8|\vec{b}|$

23. Find the equation of plane passing through the line of intersection of the planes x+y+z=6 and 2x+3y+4z-5=0 and the point (1,1,1).

24. Assume that each born child is equally likely to be a boy or girl. If a family has two children. What is the conditional probability of both are girls given that at least one is girl?

PART C

Answer any TEN questions: **10 $\times$ 3=30**

25. Show that the relation R in the set A={x/x∈z and 0≤x≤12} given by R={(a,b)/ $|a-b|$ is multiple of 4}. Is an equivalence relation?

26. Write $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

27. Find the value of x and y in $\begin{pmatrix} x+2y & 2 \\ 4 & x+y \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} = 0$ where O is null matrix.

28. If x=$\sqrt{a^{\sin^{-1}t}}$, y=$\sqrt{a^{\cos^{-1}t}}$ Show that $\frac{dy}{dx} = -\frac{y}{x}$.

29. Verify Rolle's theorem for the function f(x)=x^2+2x-8,x∈[-4,2].

30. Find the equation of tangent and normal to the curve y=x^4-6x^3+13x^2-10x+5 at (0,5).

31. Integrate: $\frac{\sin x}{\sin(a+x)}$ with respect to x.

32. Evaluate $\int_1^5 (x+1)dx$ as a limit of sum.

33. Find the area of the region founded by y^2=9x, x=2 ,x=4 and the X-axis in the I quadrant.

34. Find the equation of the curve through the point (-2,3) given that the slope of the tangent at any point (x,y) is $\frac{2x}{y^2}$.

35. Find the area of triangle with vertices A(1,1,2), B(2,3,5), C(1,5,5).

36. Prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

37. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane 10x+2y-11z=3.

38. Two groups are competing for the position on the board of directors of a corporation. The probability of I and II groups will win are 0.6 and 0.4 respectively. Further, if I group wins,the probability of introducing a new product if 0.7 and corresponding probability is 0.3 if the II group wins. Find the probability that new product introduced was by the II group.

PART D

Answer any SIX questions: **6 × 5=30**

39. Consider f:R→R defined by f(x)=4x+3. Show that f is invertible. Find the inverse of f.

40. *If* A=$\begin{pmatrix} 2 & 3 & 4 \\ 0 & -2 & 1 \\ 3 & -1 & 2 \end{pmatrix}, B=\begin{pmatrix} 2 & 0 & -3 \\ 4 & 0 & -1 \\ 3 & 4 & 5 \end{pmatrix}$ and C=$\begin{pmatrix} 5 & 6 & 7 \\ -1 & 2 & 3 \\ 4 & -5 & 4 \end{pmatrix}$

Prove that A(BC=(AB)C

41. Solve by matrix method: 2x+y+2z=5, x-y-z=0, x+2y+3z=5.

42. If y= 5cos(logx)+7sin(logx), show that $x^2y_2 + xy_1=0$

43. Find the integral of $\frac{1}{x^2-a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2-8x+5}$.

44. Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forma a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

45. Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ by the method of integration.

46. Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x = 4x\cos ecx$, $x \neq 0$ given that y = 0 when x=$\frac{\pi}{2}$.

47. Derive the equation of a line in a space passing through two given points both in the vector and Cartesian form.

48. Five cards are drawn successively with replacement from well shuffled deck of 52 cards. What is the probability that (i) all the five card are spade. (ii) none is spade.

PART E

Answer any ONE question: **$1 \times 10 = 10$**

49. a) Maximize and Minimize Z=3x+9y subjected to the constraints.

$$x + 3y \le 60$$
$$x + y \ge 10$$
$$x = y \text{ and } x \ge 0, y \ge 0 \text{ graphically}$$ 6

b) Show that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} .= \left(1+a^2+b^2\right)^3$$

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50. a) Prove that

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx & \text{if f(x) is even and} \\ 0 & \text{if f(x) is odd} \end{cases}$$

And evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x \, dx.$ 6

b) Find all points of discontinuity of f(x), where f is defined by

$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \ge 2 \\ x^2 + 1, & \text{if } x < 2 \end{cases}$$ 4